Height and Distance adjustment uncurl due to turns

By:

Jorge Lasso

Year 2019

Gratitude to the following, for kindly accepting to review the original drafts of this whitepaper:

Walt Blake
Chales Ostwick
John Shipway
Heinz Conrad
Walter Emmering
Luis Díaz

Engineering is commonly handled with numbers. Some are variables; others are constants. A good understanding of the problem at hand, from different perspectives and points of view, is also important.

GOOD engineering always ends up being a certain balance between the different perspectives and their variables.

The point where the balance settles, is driven by the optimization of the criteria that are of importance to the "stakeholder" owners of the problem. The engineer's good engineering judgement is the thread with which ALL of this is woven.

Height and Distance adjustment uncurl due to turns

Introduction

Mechanics of turn uncurl

- Height uncurl mechanics
- Distance uncurl mechanics
  - Descriptive explanation
  - The Equation
  - Inherent Error
    - Description
    - Alternatives
    - Selected method mechanics (obtaining “A”)
- Multiple turns accounting
  - Height uncurl
  - Distance uncurl
  - Combining Distance and Height uncurl
  - Multiple turns accounting when a level-off exist between turns

Application

- Uncurl of level-off
  - Turns before the level-off segment
  - Turns during the level-off segment
    - Height uncurl adjustment for turns during the level-off segment
    - Distance uncurl adjustment for turns during the level-off segment
  - Implementation of level-off height adjustment uncurl due to turns
- Uncurl of obstacles on 2nd segment
- Uncurl of obstacles on Level-off segment
- Final Segment obstacles uncurl
- Ex2nd Segment obstacles uncurl
- Application conclusion synopsis

Introduction

When an aircraft turns, the vertical path's climb angle capability is reduced.
This reduction is commonly provided in the form of a gradient decrement as a function of bank angle. This information is provided by the aircraft manufacturers. But commonly the AFM of several manufacturers will not incorporate the gradient decrement effects directly into their path angle computed results. Commonly, AFM path results are only for straight out tracks with no turns.

It will be the AFM user's responsibility to set the means of accounting the gradient decrement reduction due to turns, to the AFM's no turn path computed results. This accounting can be done by calculating adjustments of the curls of the turns, that for example; when applied to obstacles, it will convert them into equivalent obstacles in a straight no turn track. In a sense, the obstacles as "items of concern" on the track with turns, are uncurled into a straight track for the AFM to be able to consider them. Performance engineers commonly address this technique as: "uncurl" or "uncurling". *A note: performance engineering folklore might sometimes also call it "uncurlment"; but "uncurlment" is not an English word.*

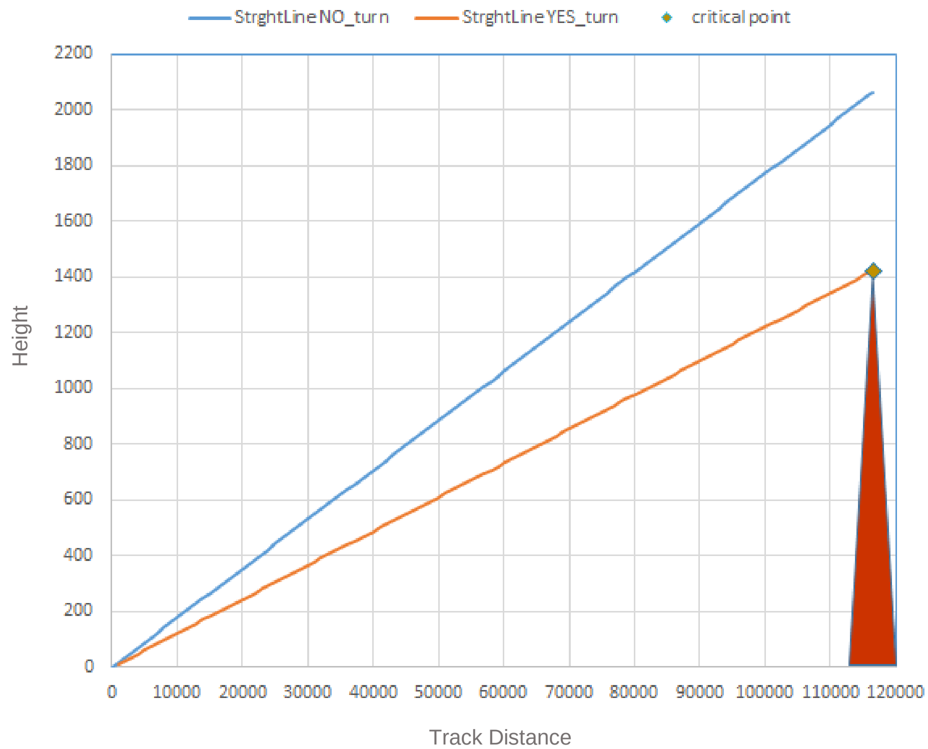
The purpose of this paper is to address the performance engineering details of this uncurl technique.

Performance engineers commonly address uncurling through a height adjustment. This paper will also consider and perhaps introduce an alternative mode of uncurling through distance adjustment.

For clarity of terms: In this paper, the vertical path that contains the actual heights that aircraft fly along a horizontal turning track will be addressed as the "YES_turn" *path. While the* "NO_turn" *path will refer to the vertical path containing heights from a horizontal track with no turns, as the AFM would depict it. Both path's heights can and will be considered in single plots against the track distance of each. And in some other particular plots; against the time of each.*

Mechanics of turn uncurl
> Height uncurl mechanics

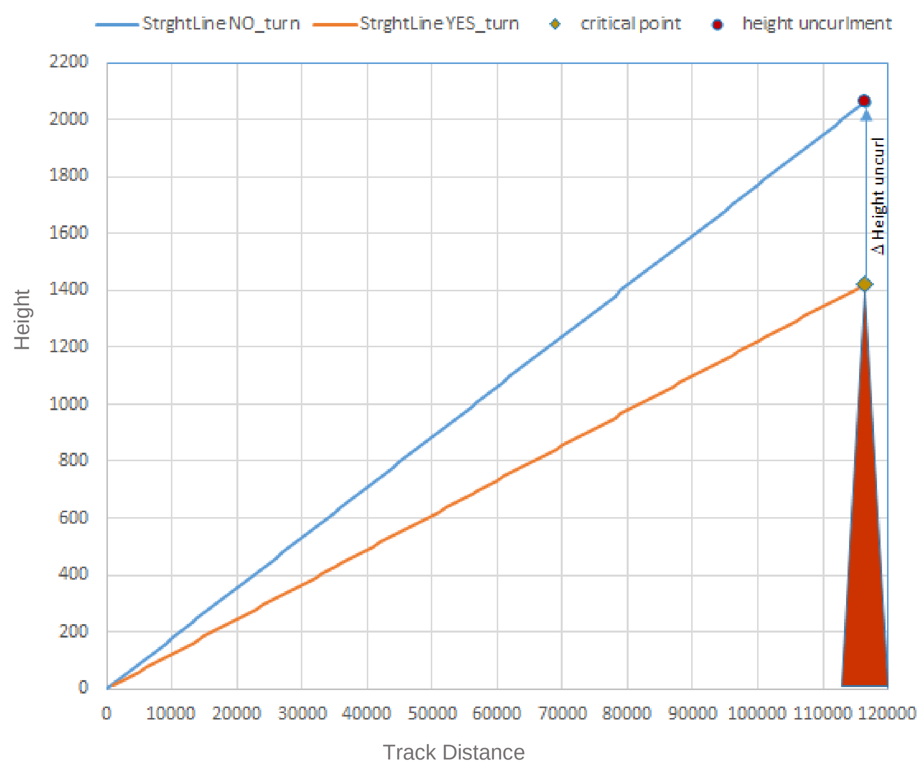
note: the flight path lines are to represent obstacle clearance paths (NET-35ft).



Δ Height uncurl = $\Delta ht_{uncurl} = \Delta Xt\,(GrdDec)$

where: "ΔXt"= Turning track distance (measured from turn start to obstacle location).
"GrdDec" = Gradient Decrement due to turning.

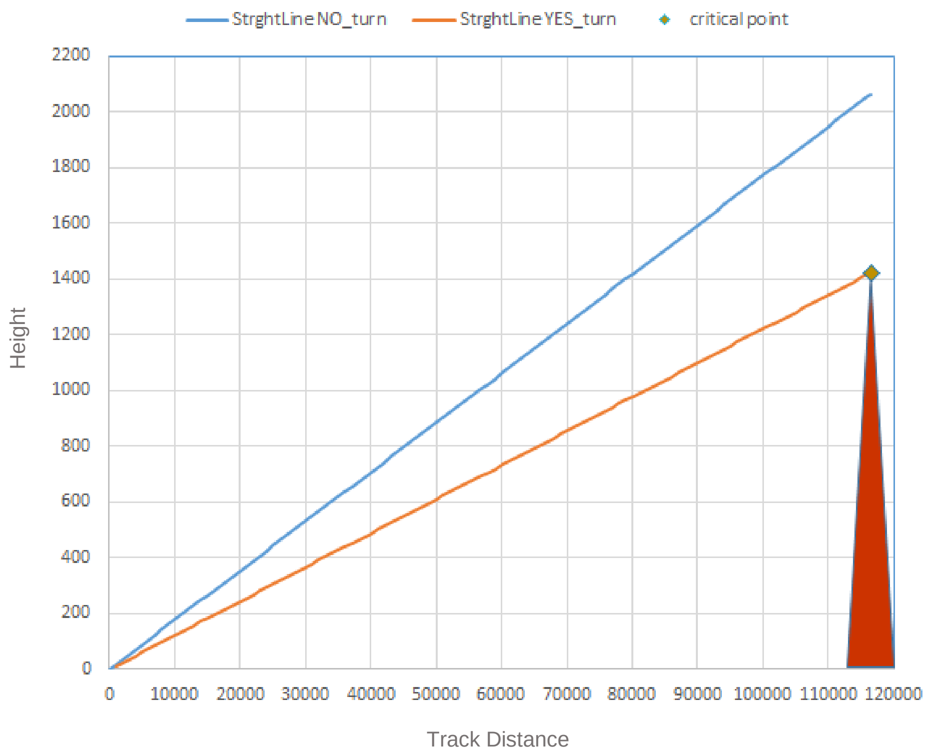
notes for ΔXt: The "t" in the "Xt" terms, stands for the axis being "turning" track distances.
"Δ" because this is NOT cumulative turning track distance.

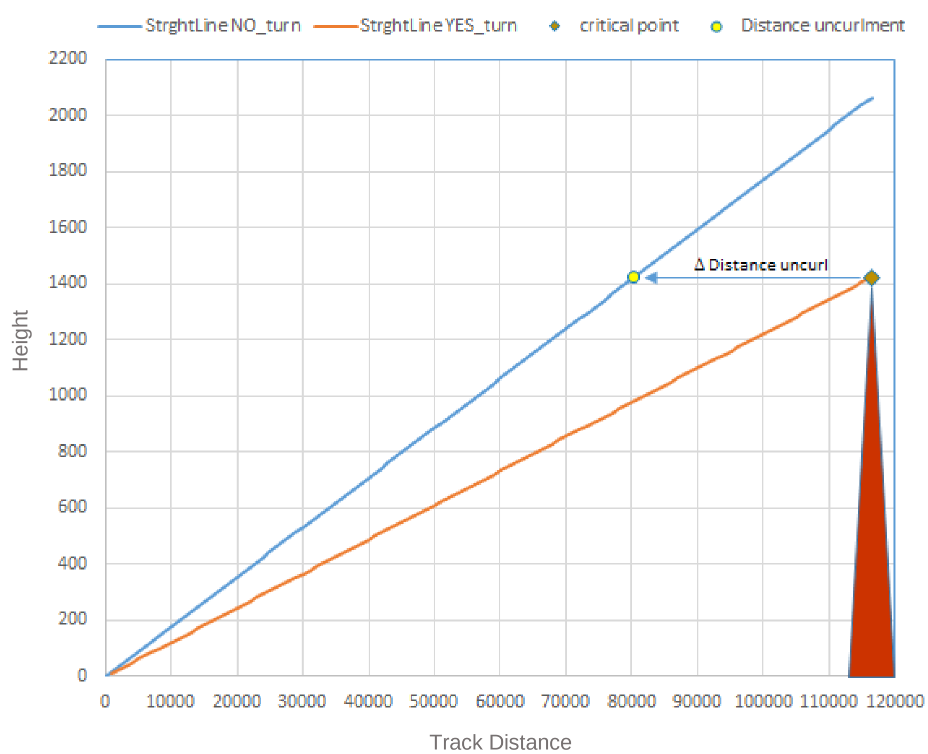


Mechanics of turn uncurl
> Distance uncurl mechanics
>> Descriptive explanation

Base Model ASSUMPTION: straight line Flight Paths

note: later on, we'll cover for possible errors of this assumption, with an error analysis.



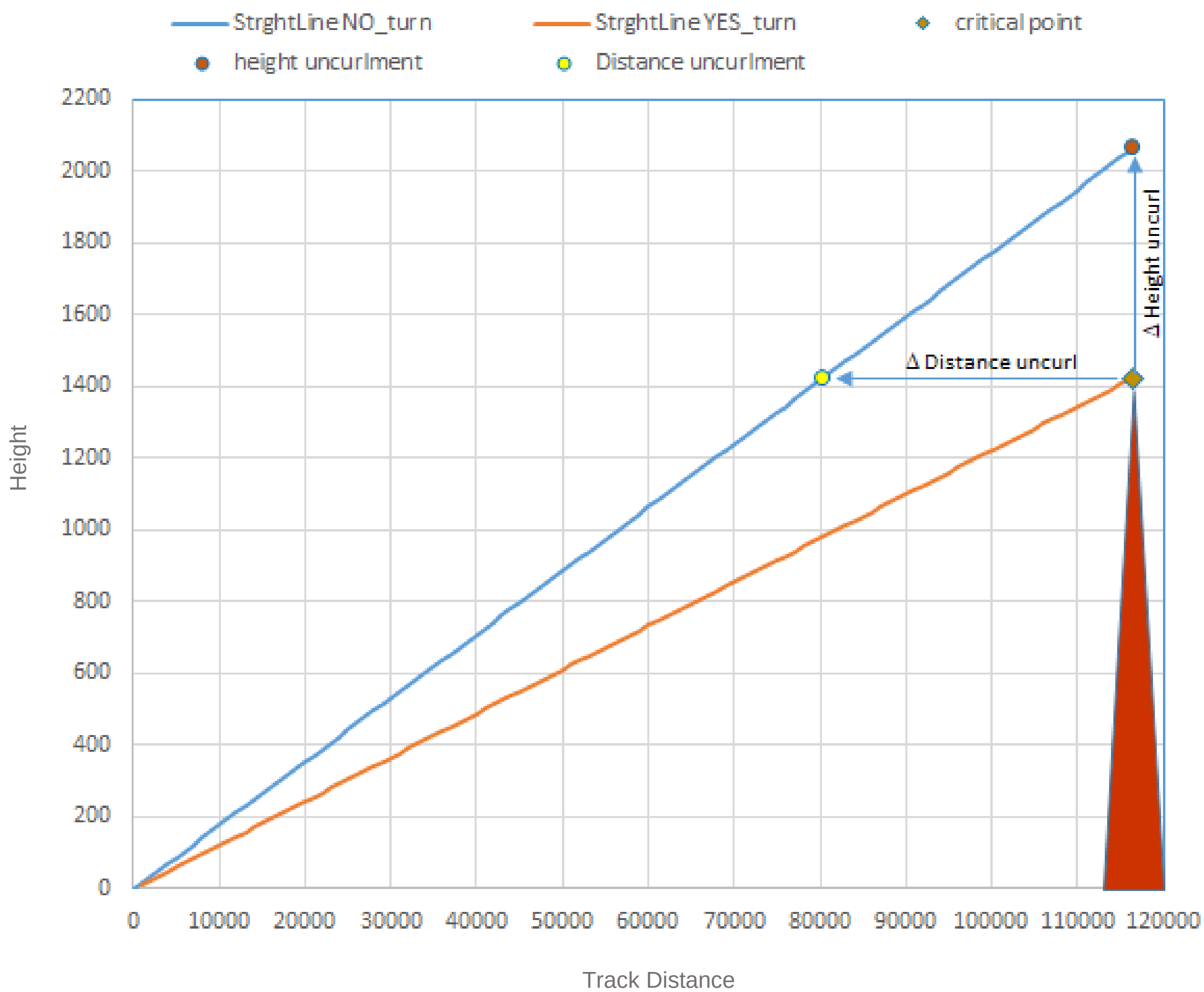


So an obstacle could be brought from the NO_turn line to the YES_turn line by either:

1/ a height increment of the obstacle at the same distance location (height uncurl)

-or-

2/ a distance location reduction of the obstacle at the same height (distance uncurl)



FACT: height uncurl will make the AFM perform obstacle clearance calculations at higher Heights than the aircraft will in reality be flying at, when over the obstacle.

IF making the AFM perform calculations at higher Heights, generates new cumbersome problems;

THEN a distance uncurl might prove convenient.

THE VALUE: <u>having the analysis alternative</u>.

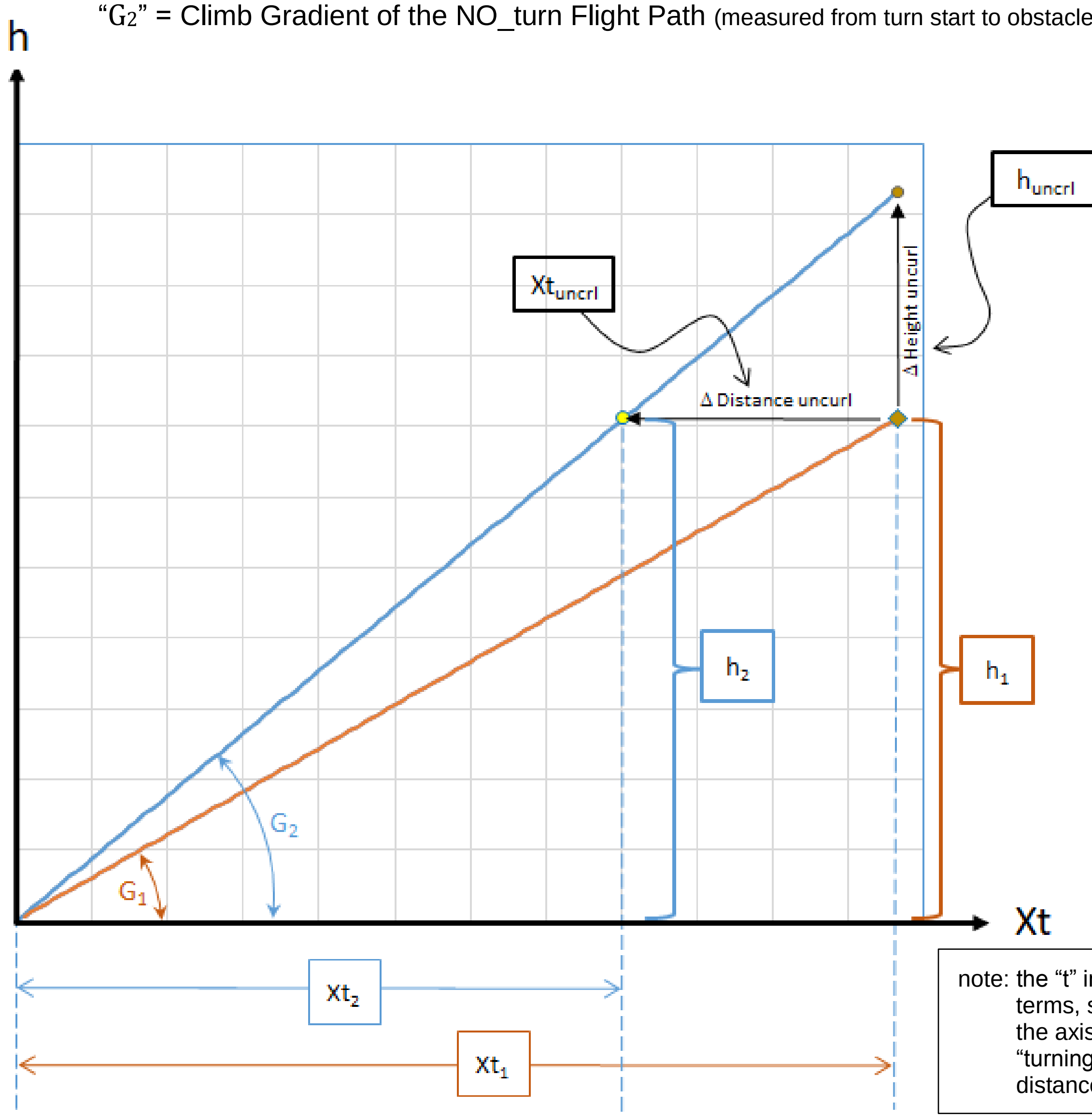
Mechanics of turn uncurl
> Distance uncurl mechanics
>> The Equation

From math trigonometry:

$$Xt_{uncurl} = Xt_1 \left(\frac{GrdDec}{G_2}\right)$$

note: math proof in annex 1

where: "Xt_{uncurl}"= Uncurl distance adjustment to be subtracted from the actual obstacle location.
"Xt_1"= Turning track distance (measured from turn start to obstacle location).
"GrdDec" = Gradient Decrement due to turning.
"G_2" = Climb Gradient of the NO_turn Flight Path (measured from turn start to obstacle location).



note: the "t" in the "Xt" terms, stands for the axis being "turning" track distance

The simplest way to grasp the essence of the above equation is to recognize that the gradient G_2 is the relation of ΔHeight / ΔDistance in the NO_turn path line. So from any point in the YES_turn path; a ΔHeight to the NO_turn path can be converted to a ΔDistance to the NO_turn path, when dividing it by G_2. Hence, a ΔHeight uncurl defined by Xt_1(GrdDec), when divided by G_2, becomes a ΔDistance uncurl.

Note that “G_2” is the only argument that’s in addition to the ones already used for height uncurl.

Also note that since G_2 is in the denominator of the equation, that means that the smaller the NO_turn climb gradient, the bigger the distance adjustment. In consequence the AFM’s calculated most limiting NO_turn NET climb gradient (from turn start to the obstacle’s location) will be the most conservative scenario.

This Uncurled distance adjustment equation was mathematically tested. And the result did confirm an appropriate distance transfer for a NO_turn line to the YES_turn line.

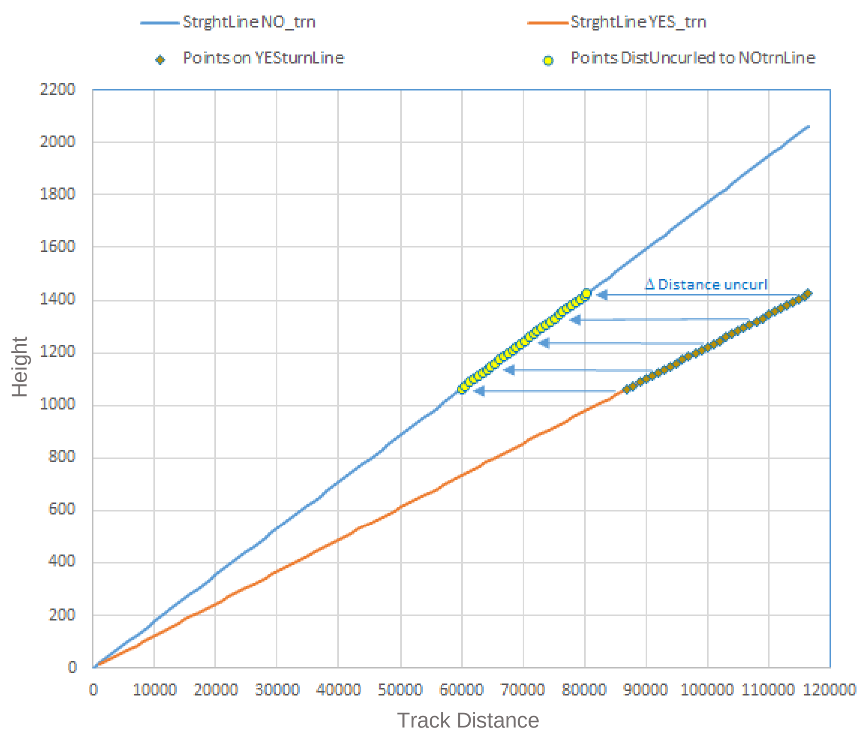
$\Delta x_{uncurl} = \Delta x_1(GrdDec/G2)$

equation test:

GradDec = 0.550%

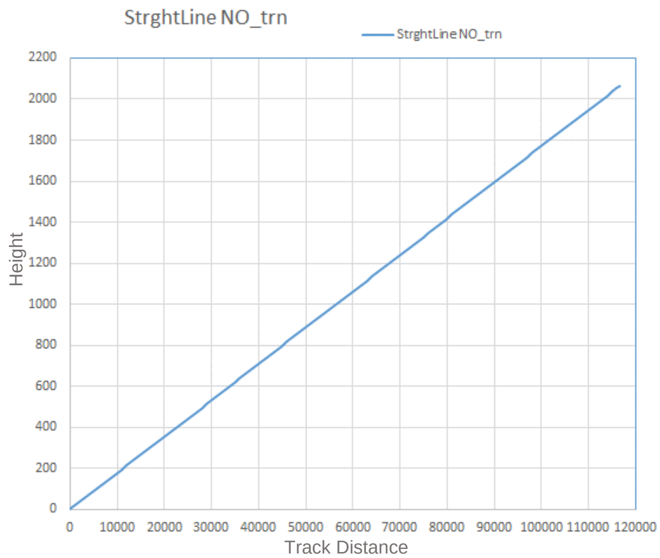
G2 = 1.7719%

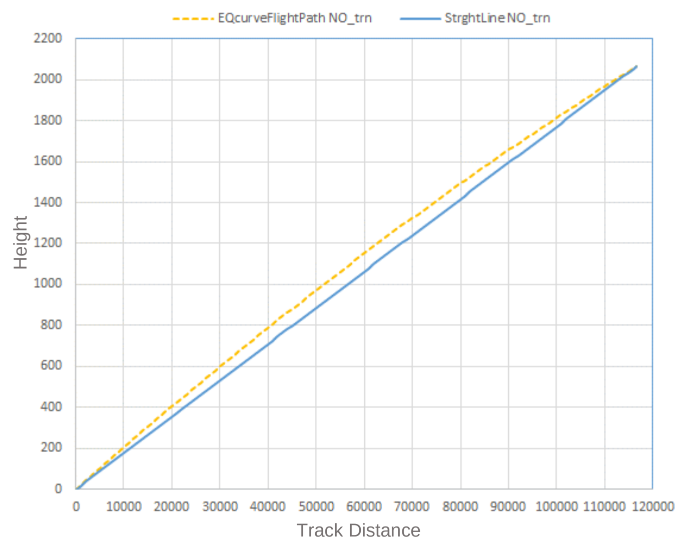
original y location	original x location	Δx_1	Δx_{uncurl}	New x location
1063.0	87000	87000	27005.5	59994.5
1075.2	88000	88000	27315.9	60684.1
1087.5	89000	89000	27626.3	61373.7
1099.7	90000	90000	27936.7	62063.3
1111.9	91000	91000	28247.1	62752.9
1124.1	92000	92000	28557.5	63442.5
1136.3	93000	93000	28868.0	64132.0
1148.5	94000	94000	29178.4	64821.6
1160.8	95000	95000	29488.8	65511.2
1173.0	96000	96000	29799.2	66200.8
1185.2	97000	97000	30109.6	66890.4
1197.4	98000	98000	30420.0	67580.0
1209.6	99000	99000	30730.4	68269.6
1221.9	100000	100000	31040.8	68959.2
1234.1	101000	101000	31351.2	69648.8
1246.3	102000	102000	31661.6	70338.4
1258.5	103000	103000	31972.0	71028.0
1270.7	104000	104000	32282.4	71717.6
1283.0	105000	105000	32592.9	72407.1
1295.2	106000	106000	32903.3	73096.7
1307.4	107000	107000	33213.7	73786.3
1319.6	108000	108000	33524.1	74475.9
1331.8	109000	109000	33834.5	75165.5
1344.0	110000	110000	34144.9	75855.1
1356.3	111000	111000	34455.3	76544.7
1368.5	112000	112000	34765.7	77234.3
1380.7	113000	113000	35076.1	77923.9
1392.9	114000	114000	35386.5	78613.5
1405.1	115000	115000	35696.9	79303.1
1417.4	116000	116000	36007.3	79992.7
1423.5	116500	116500	36162.5	80337.5



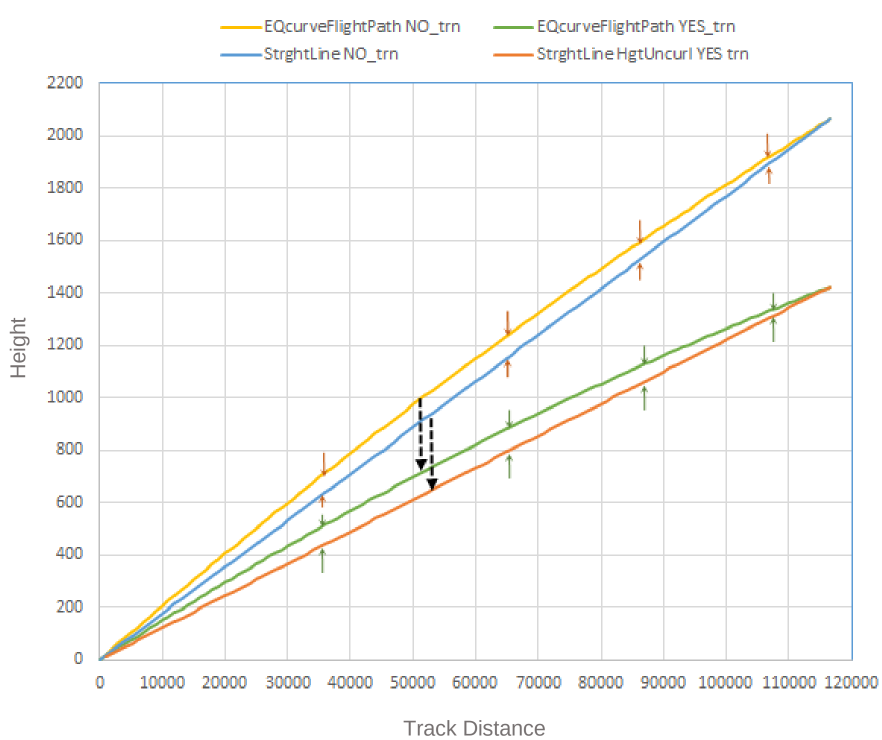
Mechanics of turn uncurl
> Distance uncurl mechanics
>> Inherent Error
>>> Description

The base model assumption for the uncurled distance adjustment equation is that the Flight Paths are straight lines.
It's recognized that actual Flight Paths are not truly a straight line but rather a curved path above a straight line from the turn start to the obstacle location.



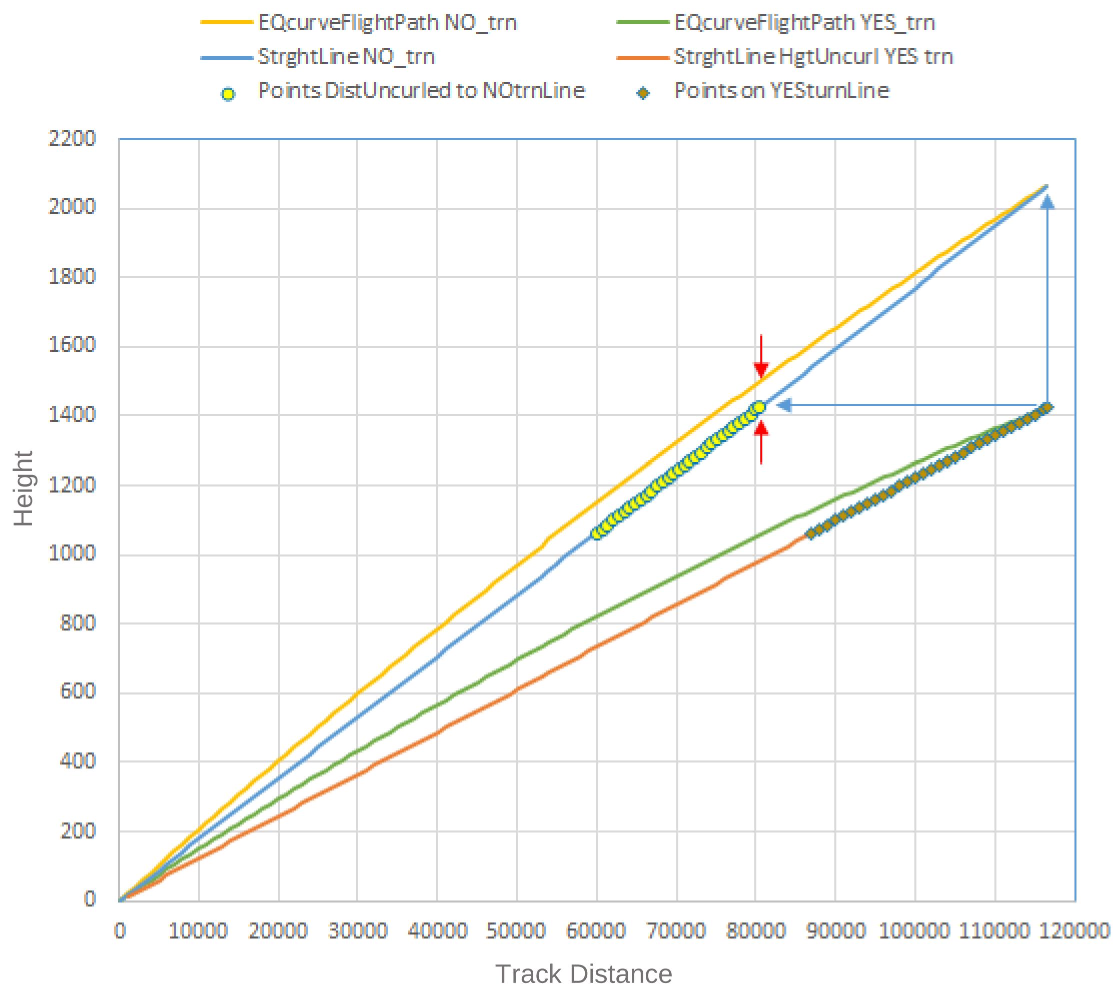


This difference will exist in both the NO_turn and in the YES_turn flight paths.
Given a constant GrdDec value between the YES_turn and the NO_turn, bear in mind that the [NO_turn to YES_turn GrdDec] applicable to the straight lines between the turn start to the obstacle location will be the same [NO_turn to YES_turn GrdDec] applicable to the Actual Flight paths. In consequence the differences between the straight line and the Actual Flight path will be the same difference in the NO_turn and YES_turn paths, at any specific track distance location.



So,.. when attempting to uncurl an obstacle from the YES_turn actual Flight path to the AFM's NO_turn flight path with the above resolved Uncurled distance adjustment equation,…

<u>an inherent error in height will YES exist</u>!



This error needs to be accounted.

Mechanics of turn uncurl
> Distance uncurl mechanics
>> Inherent Error
>>> Alternatives

Three alternatives were considered:
1- To calculate the exact error value.
2- To assume a very conservative error value.
3- To calculate an approximate error with appropriate conservatism.

Alternative -1- (To calculate the exact error value), was given extensive, significant thought by this author. And from this thought process it was concluded that the calculation of an exact error value would entail such a complexity that it might make it impractical.

Alternative -2- (To assume a very conservative error value) was considered. The intent would be to devise a conservative high value of height increment error, that the user feels comfortable as safe, and always use this value. This alternative could represent the simplest implementation. But at a cost. It might be in many instances over conservative. But it might be practical, hence this author still valued this alternative as the second best choice.

Alternative -3- (To calculate an approximate error with appropriate conservatism) was the preferred choice of this author. In this alternative the height error to be added is calculated in association to the specific track distance from the turn start to the obstacle location. The conservative assumptions to be incorporated in this calculations are:

1/ The biggest error in height between the straight line and the actual flight path, will be assumed as the actual error.

2/ The statistical most conservative constants are chosen for defining the deviation of the actual flight path from a straight line path.

These should lead to a conservative error height value that should not be excessively conservative as it still remains specific to the actual turning track distance (turn start to obstacle location distance).

For the chosen alternative -3- the following equation is mathematically derived for error height adjustment (Δh_E):

$$\Delta h_E = \frac{-A(Xt_1)^2}{4}$$

note: math proof in annex 1

where: "Δh_E"= Error height adjustment to be added to the obstacle's height.
"A" = Statistical constant modeling the actual flight path as a parabola of equation: Flight path height = $Ax^2 + Bx + 0$
"Xt_1"= Turning track distance (measured from turn start to obstacle location).

Mechanics of turn uncurl
> Distance uncurl mechanics
>>Inherent Error
>>>Selected method mechanics (obtaining “A”)

As the description of “A” for error height equation indicated, the actual flight path is to be modeled with a parabola of equation: height = $Ax^2 + Bx + 0$.

where “x” = track distance. And “A” and “B” are the constants of the quadratic equation.

If the model is to represent a GROSS height, then for a two engine aircraft, the conversion from GROSS to NET height at each “x” track distance location would be: GROSS height – (0.008)x.
So the constants could be calculated for an actual flight path as being the GROSS path, and the conversion to NET path would consist in only subtracting 0.008 to the “B” constant. Furthermore the “A” constant would remain unchanged.

Calling programs to SCAP Climbout routines could conveniently be used to calculate actual flight path for a variety of conditions. This author did just that for a 737-700Wglt with CFM56-7B22.

Input variables combinations considered:

Pressure Altitudes: 0ft, 2000ft, 4000ft, 6000ft, 8000ft, 10000ft.
Weights: 150000Lb, 140000Lb, 130000Lb, 120000Lb.
Temperatures: 0°C, 10°C, 20°C, 30°C, 40°C
Flaps: 1, 5, 10, 15, 25, UP

Notes:
Palt, Wgt, and Temp are at the start of the evaluated flight path.
Climb is maintained up to 3000ft above the flight path start Palt.
Only results with initial climb gradients above 1.2% were considered.
AFM’s VREF+70 was considered for Flaps UP (VREF+70 = final climb speed).
V2 was considered for all flaps that are not Flaps UP.

With Excel spreadsheet’s available “lest square curve fitting method” of approximation, the constant for each combination was gathered. From the error height adjustment equation ($\Delta h_E = \frac{-A(Xt_1)^2}{4}$), recognize that the numerator has a negative value sign. In consequence the smaller the value of “A”, the bigger the value of Δh_E will be. Higher values of Δh_E will serve appropriately more conservative. In consequence our desire is for the statistical minimum values of “A”.

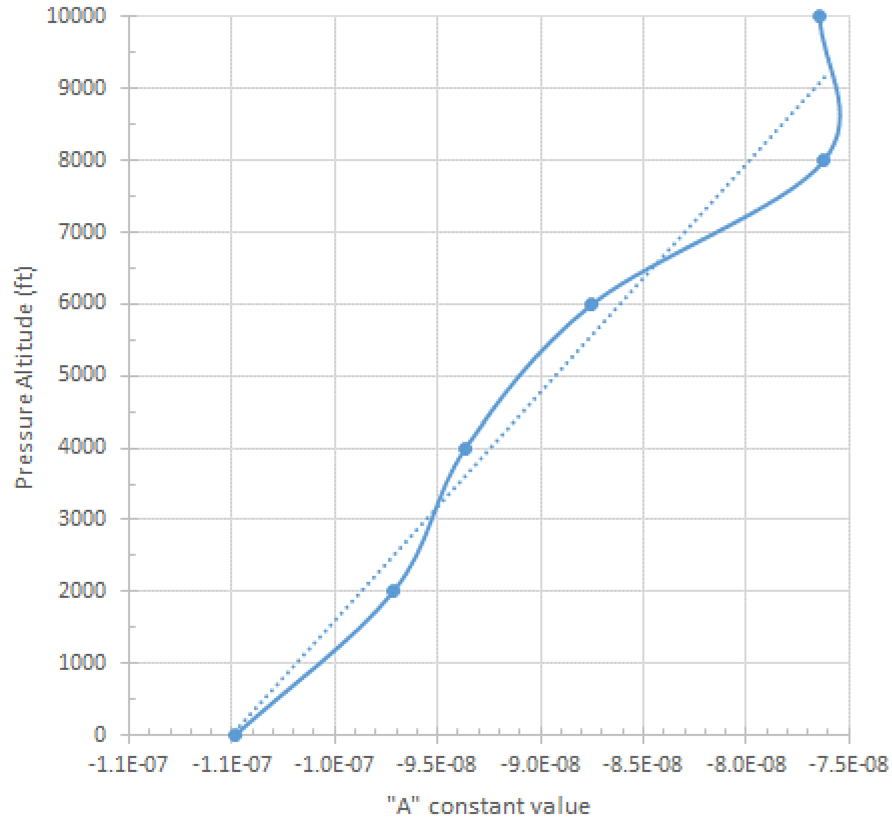
The results summary follow:

Parabolic Flt Path aproximation: height "h" = $Ax^2 + Bx + 0$

Palt (ft)	minimum "A"	-GROSS- "B"	Gross-NET	maximum -NET- "B"	average r^2
0	-1.04853E-07	0.077596471	0.80%	0.069596471	0.999999745
2000	-9.71607E-08	0.072705128	0.80%	0.064705128	0.99999973
4000	-9.36458E-08	0.066458615	0.80%	0.058458615	0.99999918
6000	-8.74837E-08	0.061359703	0.80%	0.053359703	0.999997804
8000	-7.62314E-08	0.054947911	0.80%	0.046947911	0.999999366
10000	-7.64215E-08	0.048806266	0.80%	0.040806266	0.999996772

note: "r^2" = matching accuracy of the model. The closeer to 1.0 the more accurate the match.

The minimum “A” are plotted.



From the above plot, two options are judged reasonable for determining “A”.

1st option: To only select the smallest minimum of -1.0E-07 (from SL) and apply it to all altitudes.
2nd option: So have and use straight line approximation of the above graph.

To evaluate the options, a 180 degree heading change climbing turn is considered:

$$\Delta h_E = \frac{-A(Xt_1)^2}{4}$$

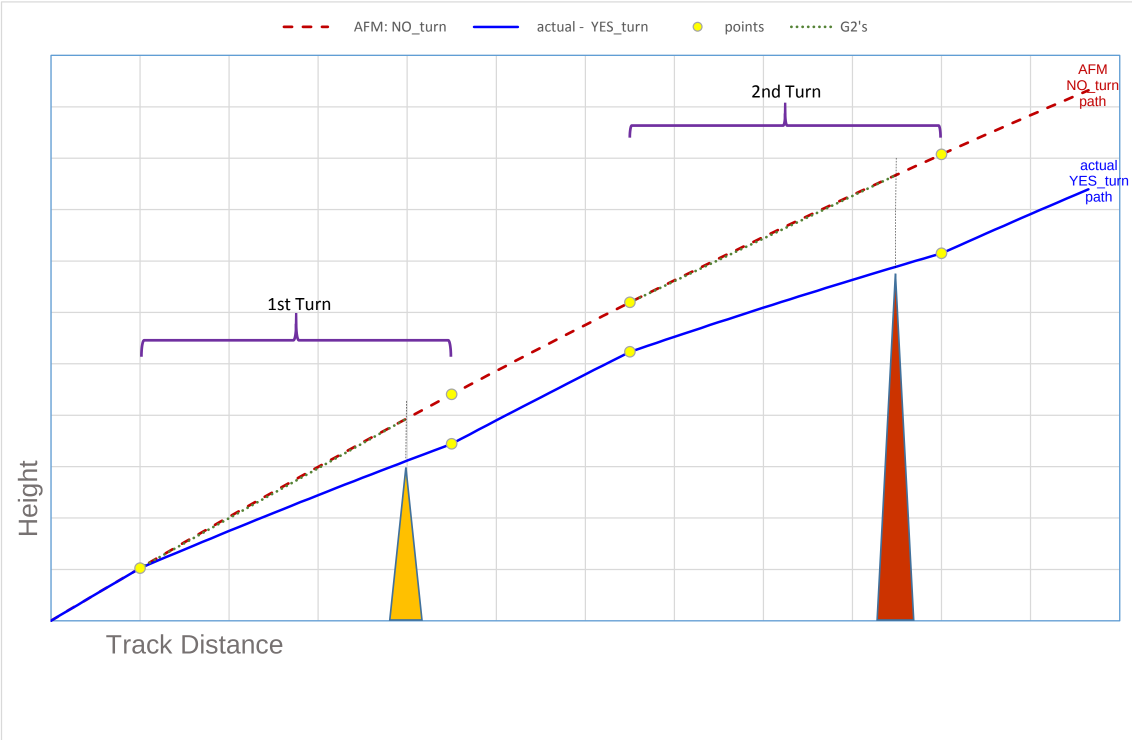
2000ft Palt 737-700Wglt 180° turn	Track distance from Turn start to Obstacle loc. [ΔXt] (ft)	A = -1.04853E-07 [Δh_E] (ft)	-7.64215E-08 [Δh_E] (ft)
FlapUP 25deg wingBank	28100	21	15
Flap 5 15deg wingBank	12850	4	3

(note: turn track distances, calculated at speed set to corresponding flap settings (V2 for Flap5; Flinal Climb airspeed for FlapUP)

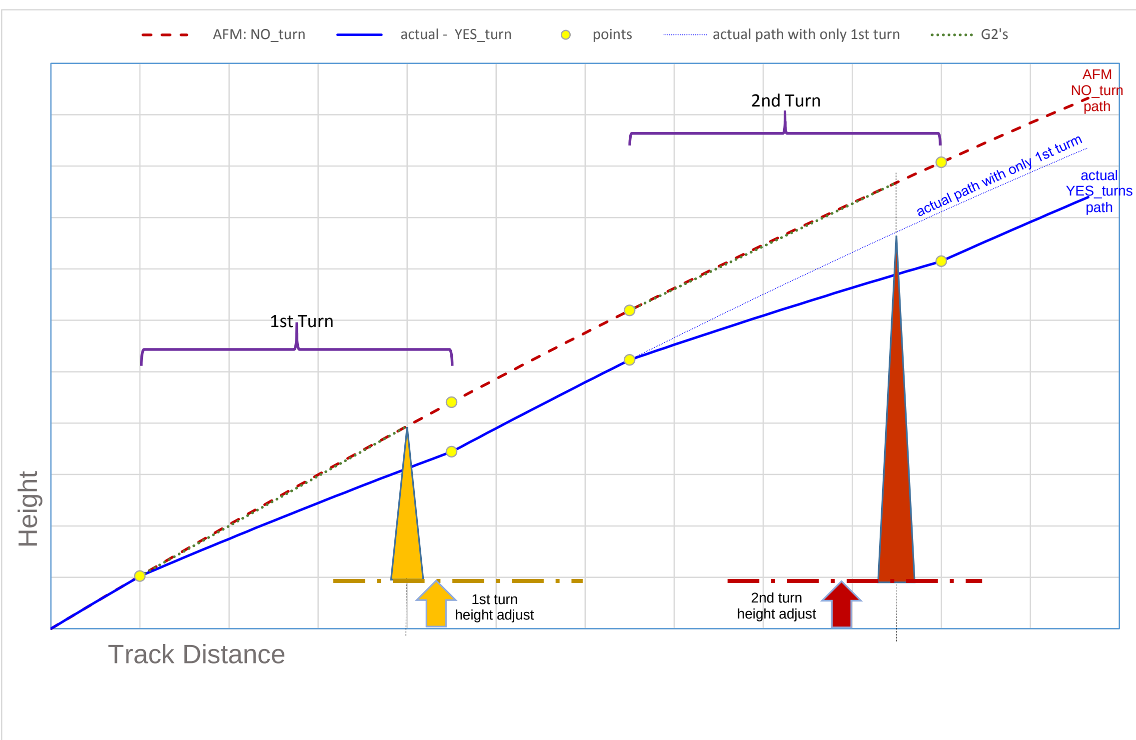
This author believes that both options serve reasonable.

Mechanics of turn uncurl
> Multiple turns accounting
>> Height uncurl

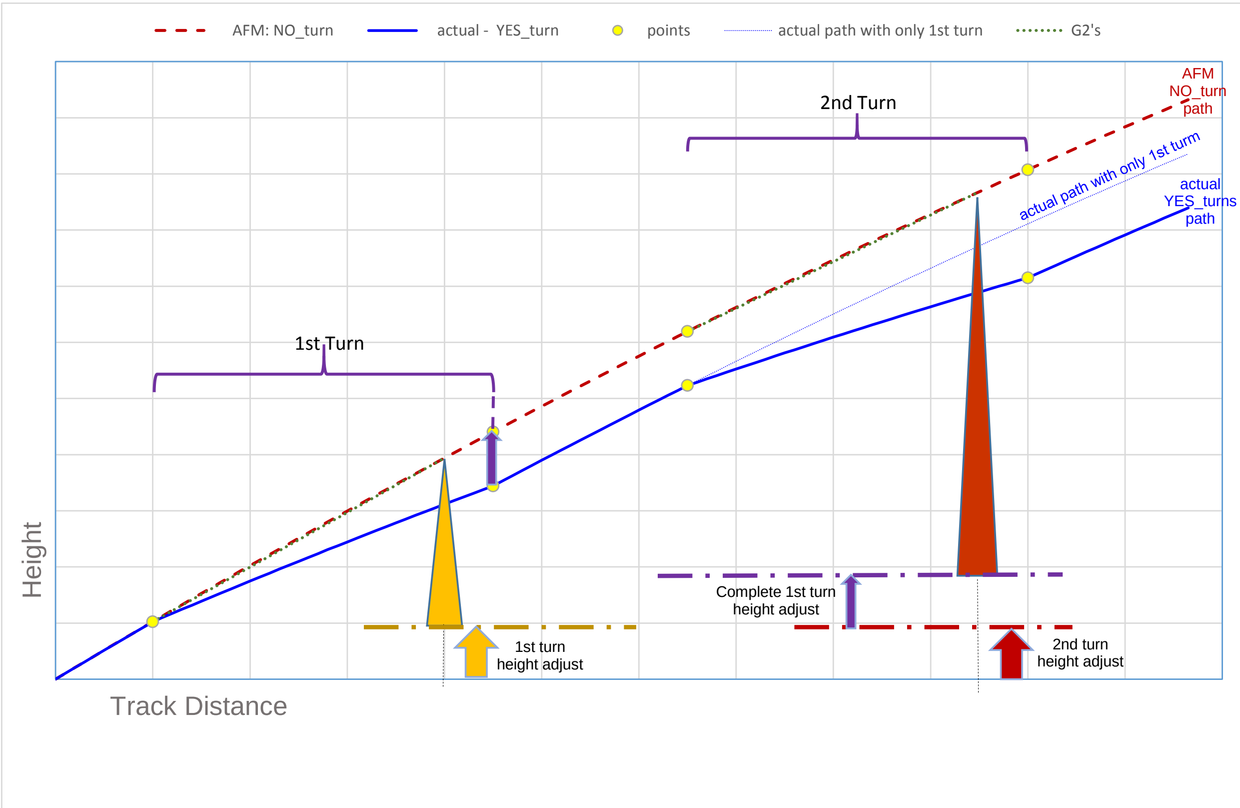
For height uncurl, it's recognized that when multiple turns exist, the height uncurl adjustment from all the <u>complete</u> turns preceding the obstacle must be accounted. They add up.



In the sketch below, the obstacle in the 1st turn correctly uncurls on the AFM NO_turn path. But, for the obstacle in the second turn; if only its 2nd turn height adjustment is applied, it will fall short of reaching the AFM NO_turn path. The 2nd turn gradient decrement will have been uncurled only to a flight path where the 1st turn gradient decrement still exist.

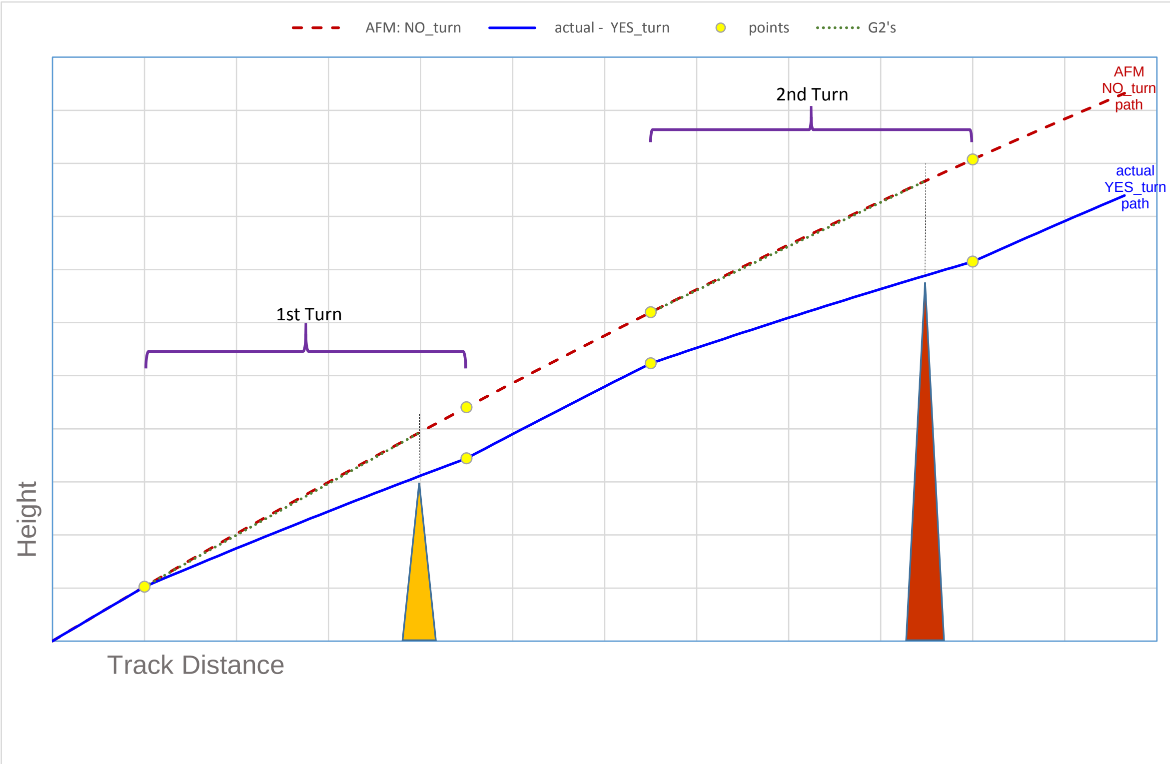


The height uncurl adjustment from preceding <u>complete</u> 1st turn must also be added in order for the obstacle top to be correctly uncurled on the AFM NO_turn path.

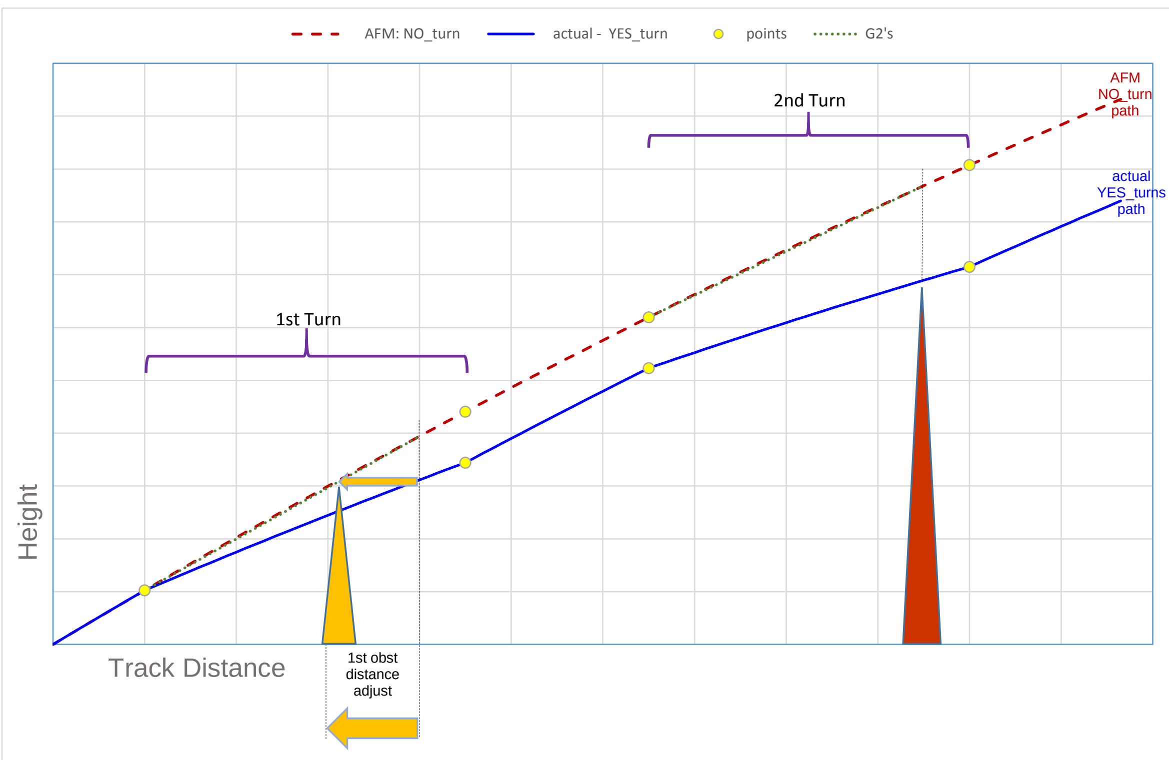


Mechanics of turn uncurl
> Multiple turns accounting
>> Distance uncurl

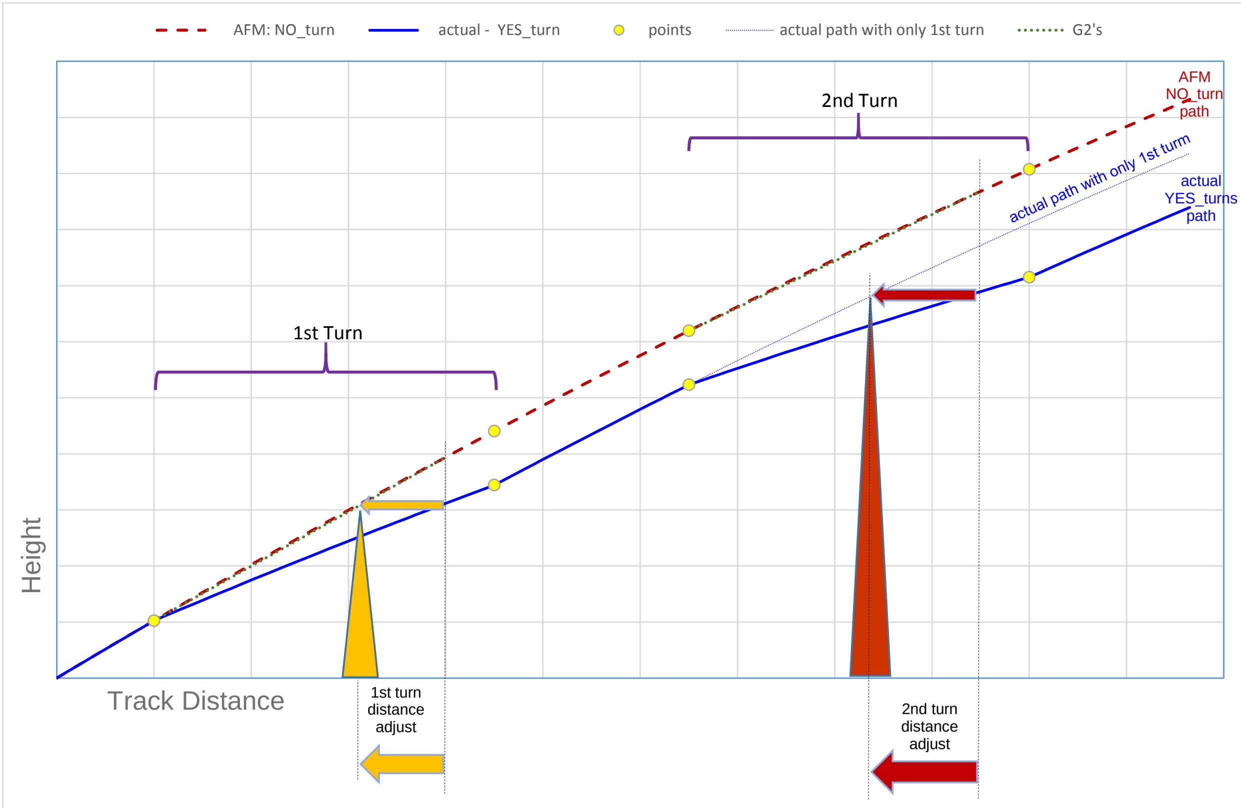
In similar fashion to height uncurl; the Distance uncurl of different turns, also add up. The distance adjustments effects from all the <u>complete</u> turns preceding the obstacles must be accounted.



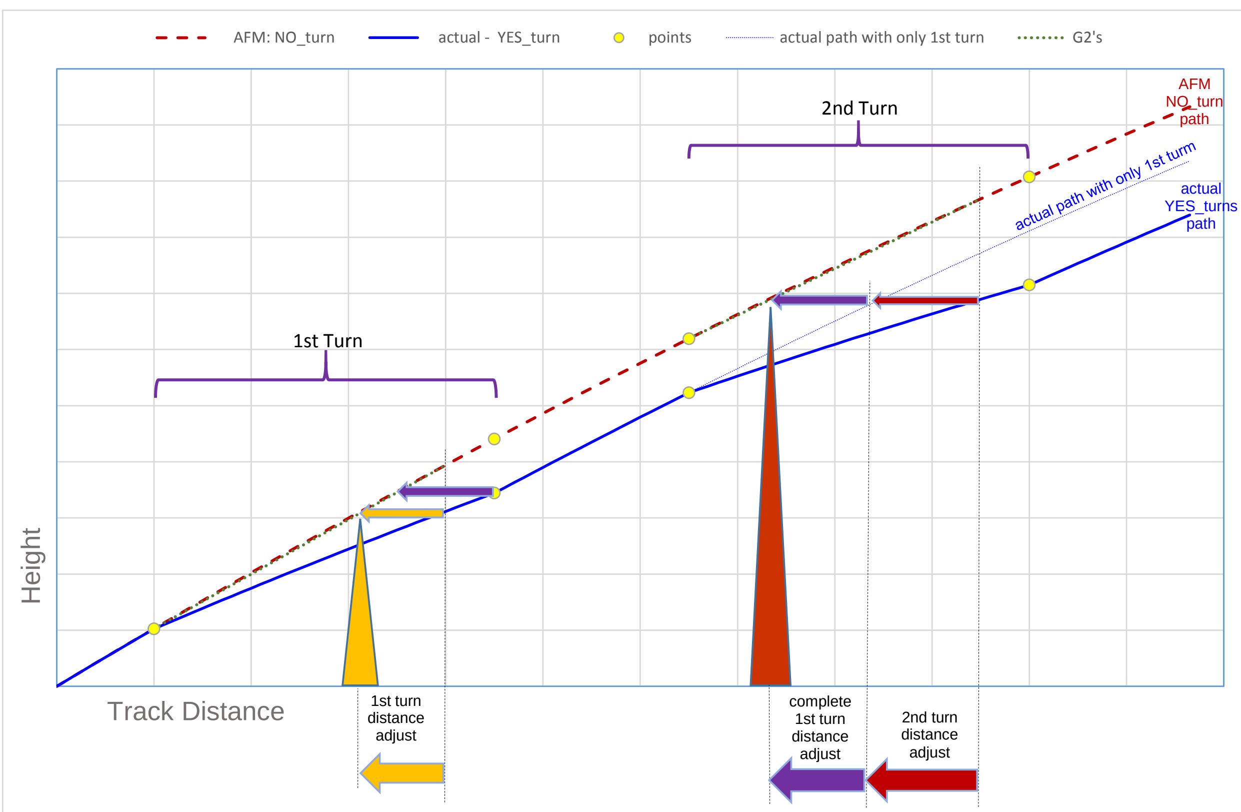
The obstacle in the 1st turn correctly uncurled on the AFM NO_turn path.



But, for the obstacle in the second turn; if only its 2nd turn height adjustment is applied, it will fall short of reaching the AFM NO_turn path. The 2nd turn gradient decrement will have been uncurled only to a flight path where the 1st turn gradient decrement still exist.



The height uncurl adjustment from preceding <u>complete</u> 1st turn must also be added in order for the obstacle top to be correctly uncurled on the AFM NO_turn path.



For distance uncurl: In a different fashion; the Error height adjustment is not additive. The error relates only to the obstacle's most recent turn. This is because the most recent turn start location, already entails accurate information of height.

Recall that the error equation is: $\Delta h_E = \frac{-A(Xt_1)^2}{4}$

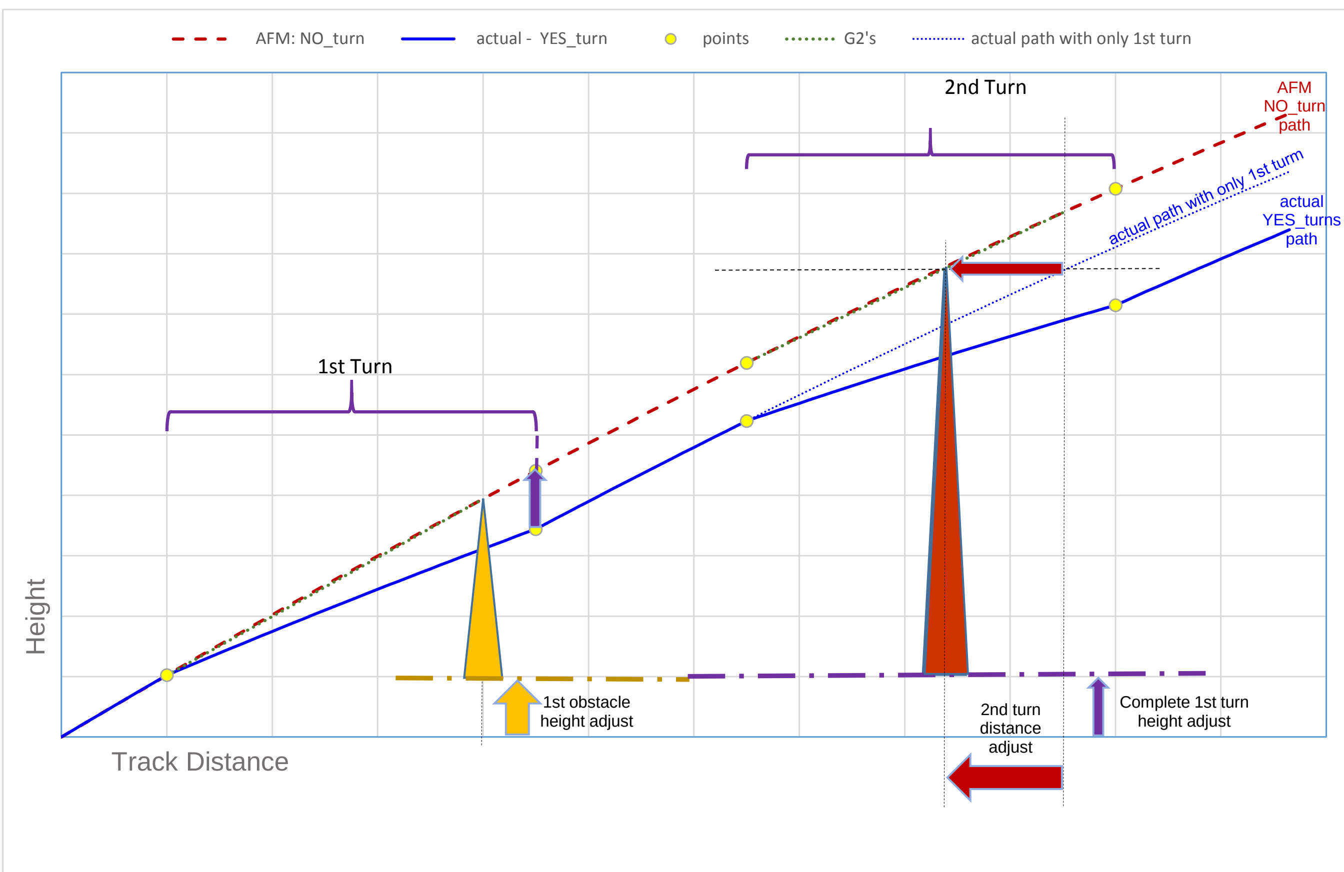
In this fashion; from the above distance uncurl examples, the "Δh_E" error calculated with "Xt_1" obtained from the 1st turn's start location up to the obstacle within the 1st turn's location, is applicable only to the obstacle within the 1st turn.

Recognize that for the 2nd turn; its start location height on the AFM NO_turn path is an accurately known value. In consequence, the "Δh_E" error calculated with "Xt_1" obtained from the 2nd turn's start location up to the obstacle within the 2nd turn's location, is applicable only to the obstacle within the 2nd turn. No additional error from the 1st turn needs to be added to the obstacle within the 2nd turn.

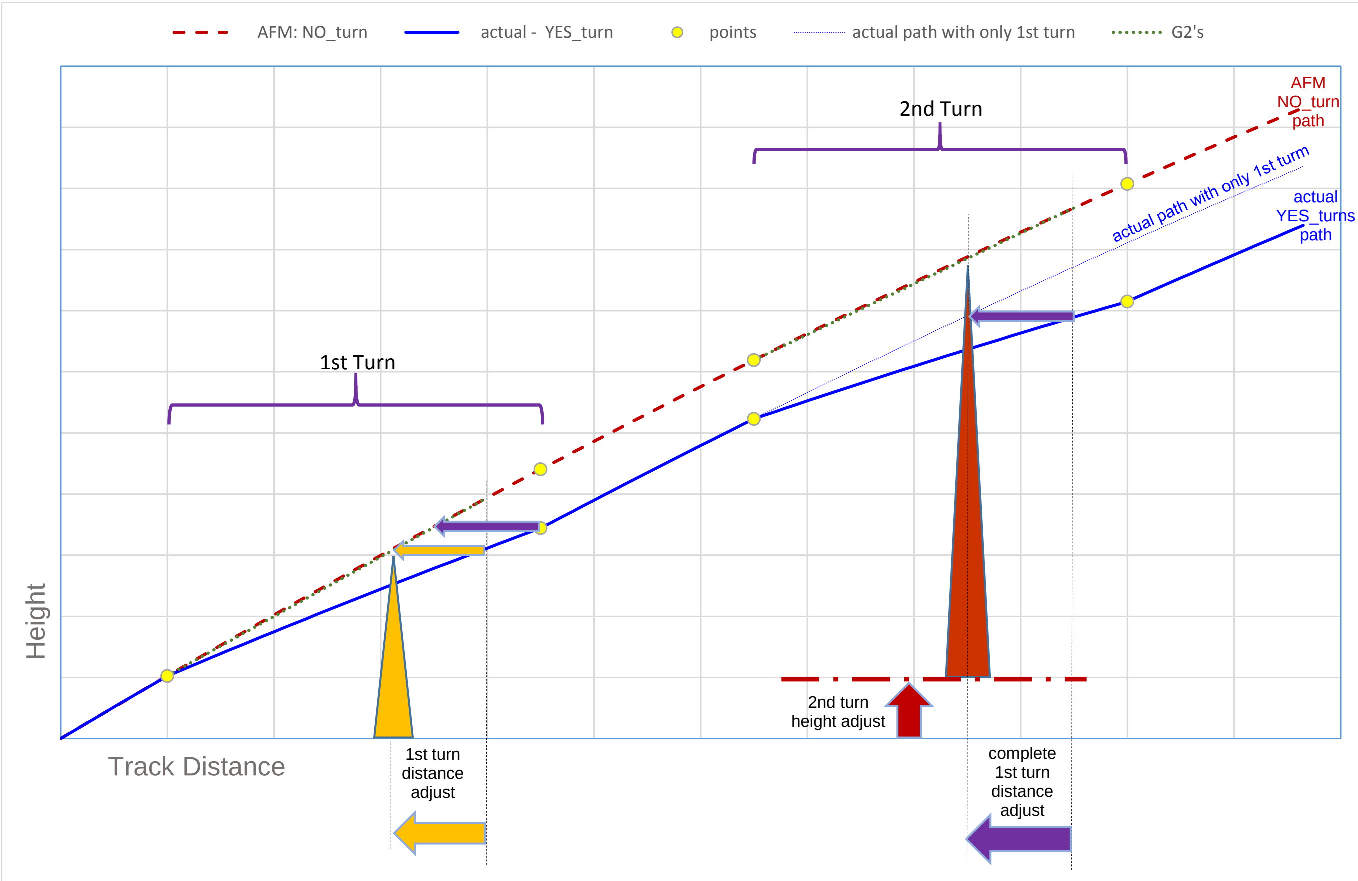
Mechanics of turn uncurl
> Multiple turns accounting
>> Combining Distance and Height uncurl

Distance adjustment uncurl, can also be combined with height adjustment uncurl on any given obstacle.

For a given obstacle, a height adjustment uncurl of a prior turn can be combined with a distance adjustment uncurl of the current turn.



In similar fashion, for a given obstacle, a distance adjustment uncurl of a prior turn can be combined with a height adjustment uncurl of the current turn.



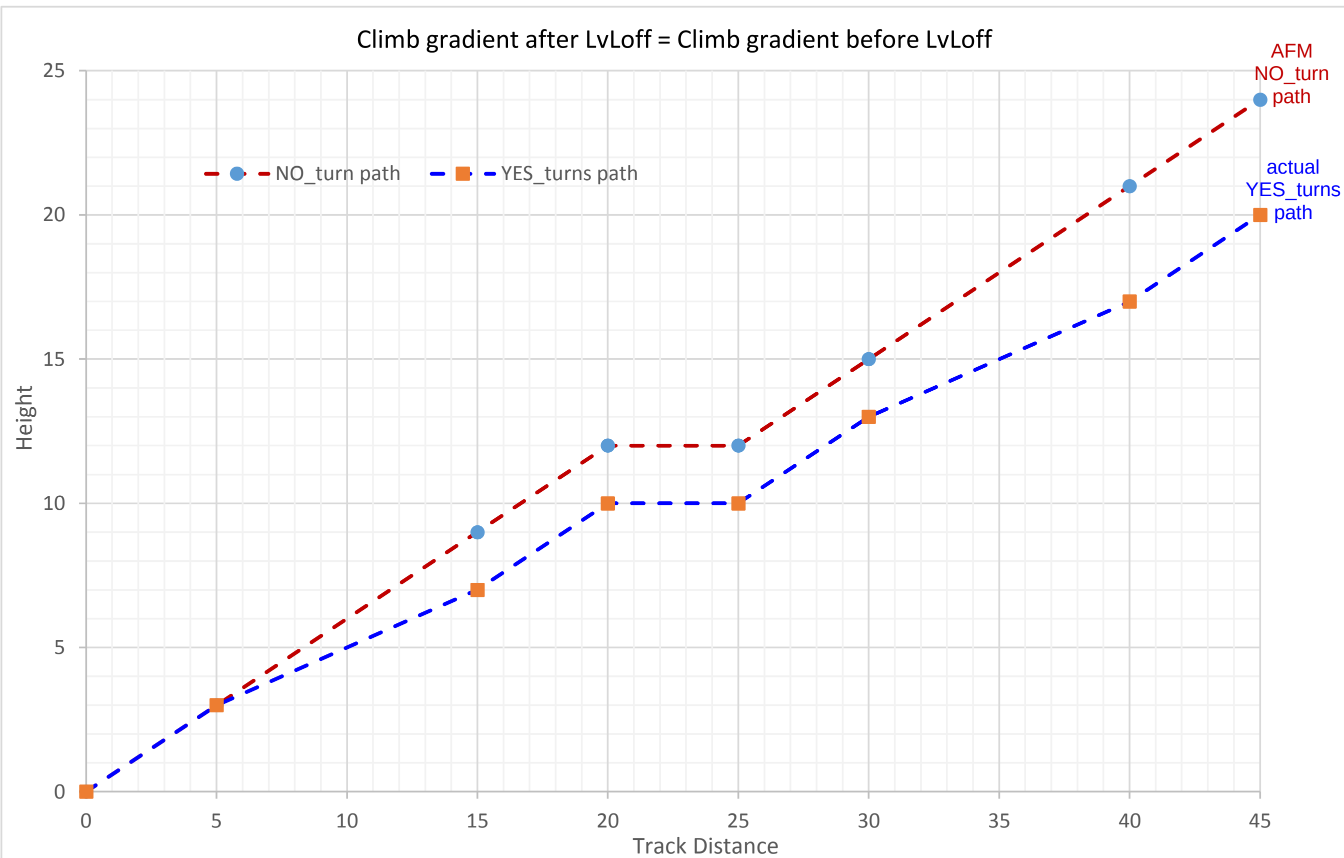
In these particular cases where height adjustment uncurl is being applied to the current turn, any or all of the Error height adjustments Δh_E from prior distance adjustments, will not apply. In other word: $\Delta h_E = 0$ (always) when the current turn is being uncurled with height adjustment.

Mechanics of turn uncurl
> Multiple turns accounting
>> Multiple turns accounting when a level-off exist between turns

When turns exist prior to the level-off, then the level-off height must be uncurled with a height adjustment. The substantiation for this will be elaborated later on this paper. For the moment; let's just recognize this as a fact.

When the above is the case; for the application of distance uncurl to obstacles existing after the level-off segment, the contribution from the turn prior to the level off might not be exact. This only affects distance adjustment uncurl; it doesn't affect height adjustment uncurl.

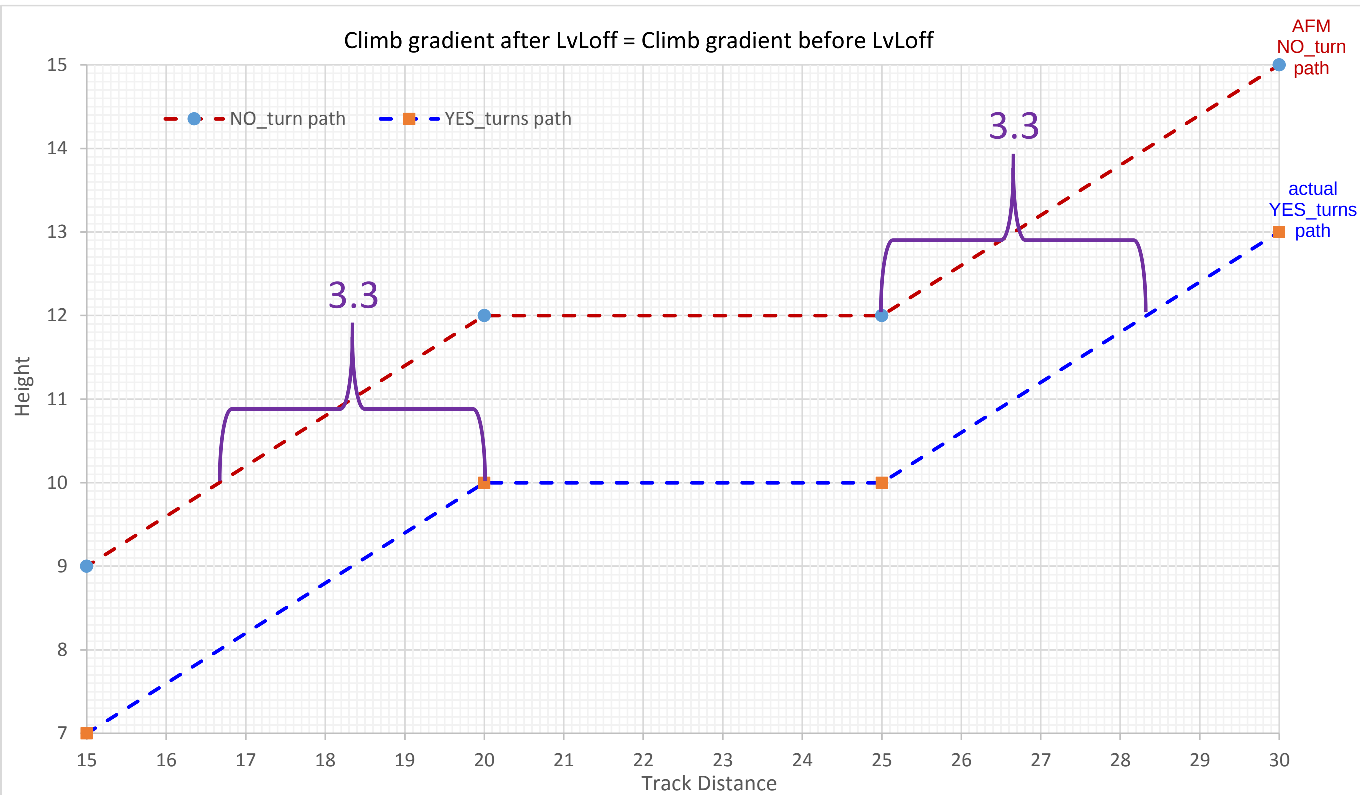
This distance uncurl inaccuracy results as a consequence of the level-off being uncurled with height. And the degree of inaccuracy will depend on the gradient after the level-off versus the gradient before the level-off. This can be graphically visualized with an exaggerated scale sample sketch with a turn before the level-off segment *(between 5 and 15 track distance units)* and a turn after the level-off segment *(between 30 and 40 track distance units)*.



The prior exaggerated scale sample sketch has been built for illustration purposes with an arbitrary 60% climb gradient before and after the turn. Climb gradient after the level-off = climb gradient before the level-off.

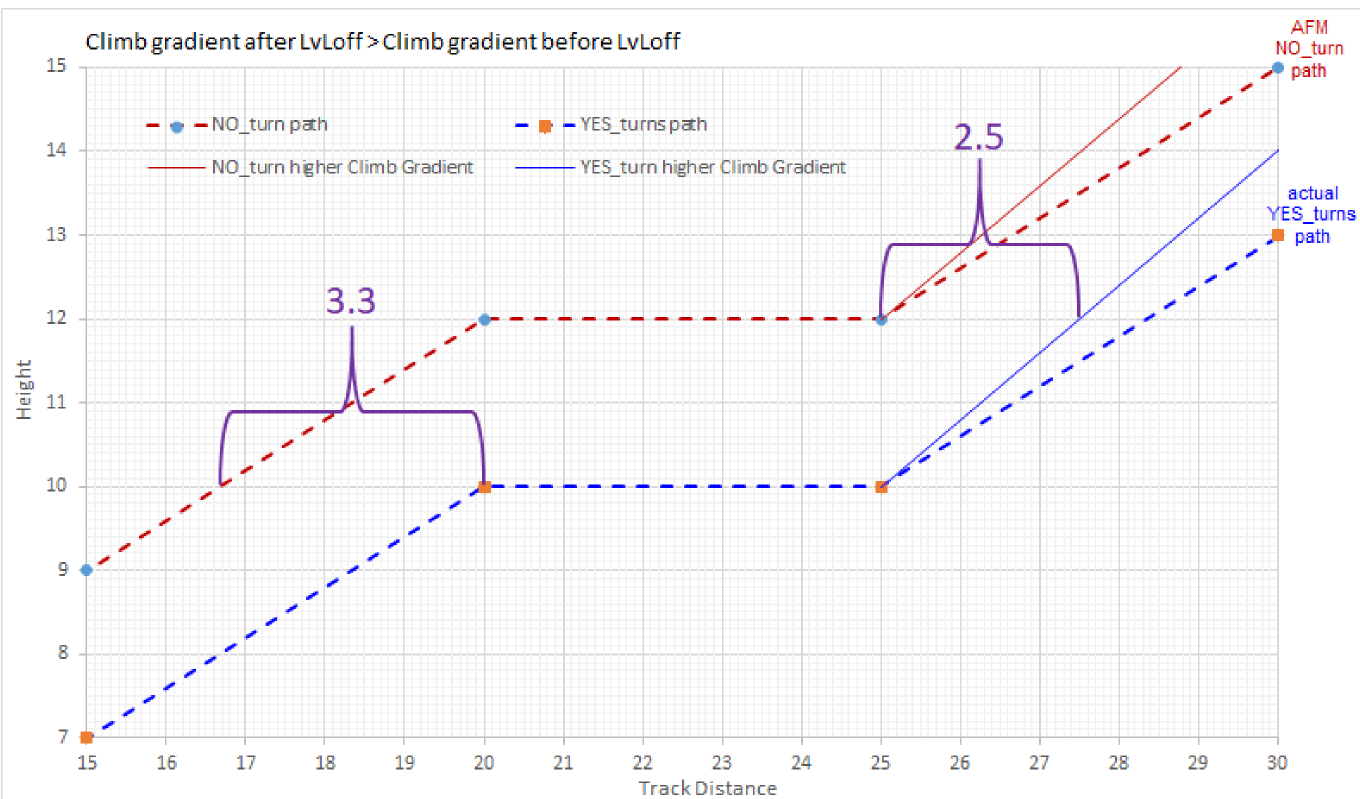
Both turns gradient loss due to turn = 20%.
The 1st turn creates a distance displacement between the NO_turn path and the YES_turn path. After the 1st turn ends, this distance displacement will prevail at the same value until the level-off segment is reached.

If and only if the gradient after the level-off segment remains the same as the gradient before the level off, then the distance displacement value from the 1st turn will maintain the same value at the end of the level-off segment to in turn be accounted as a starting additional distance displacement for the 2nd turn. By zooming into the segment between 15 and 30 track distance in the sample sketch, the above fact can be verified by measuring the distance displacement existing before the after the level-off segment.



If the climb gradient after the level-off is greater than the climb gradient before the level-off; then, by geometry, the distance displacement after the level-off will result of a lower value than the distance displacement before the level-off.

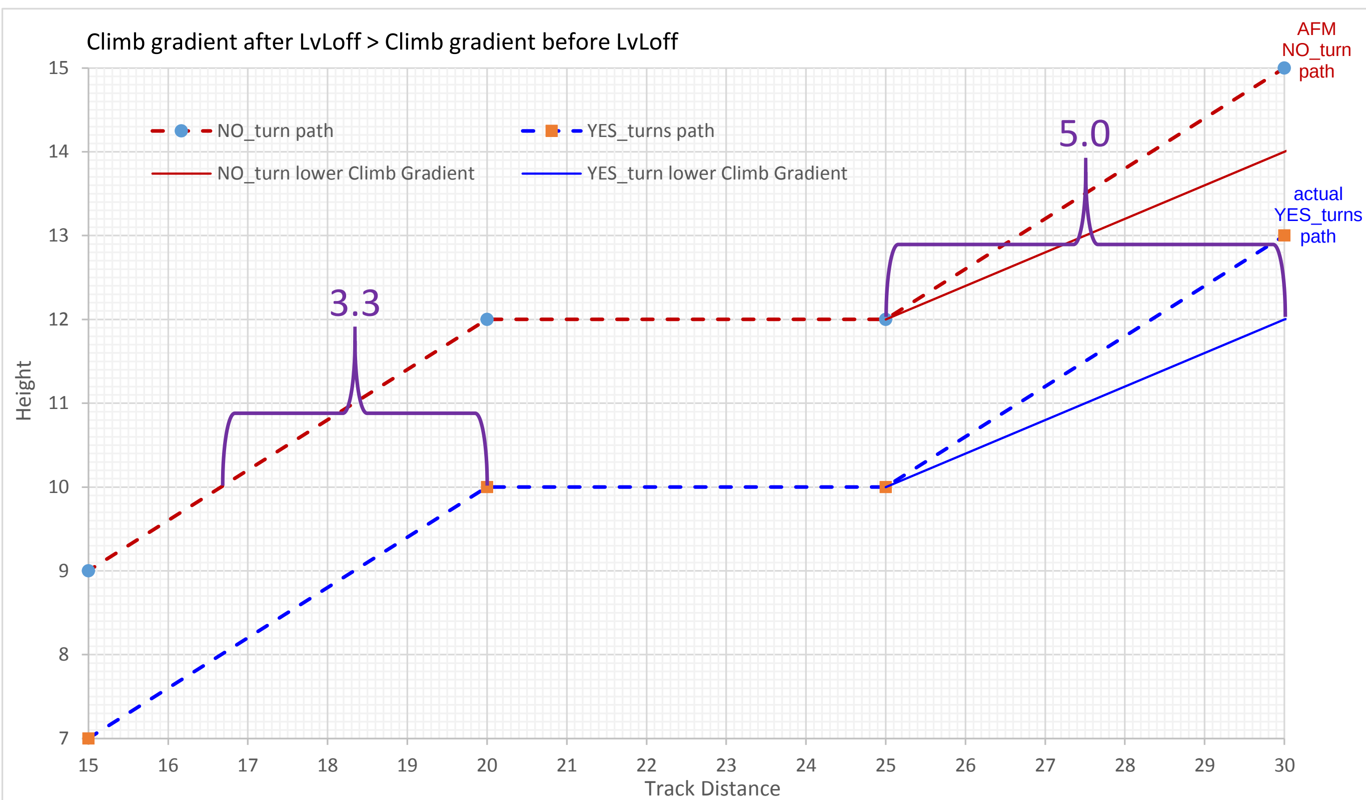
Climb Gradient BEFORE LvLoff = 60%					Turnning climb gradient loss = 20%		
Climb Gradient AFTER LvLoff = 80%							
		NO_turn path			YES_turns path		
Δx	x	y	%			y	%
	0	0		start		0	
5	5	3	60%	1st turn start		3	60%
10	15	9	60%	1st turn end		7	40%
5	20	12	60%	LvLoff start		10	60%
5	25	12	0%	LvLoff end		10	0%
5	30	16	80%	2nd turn start		14	80%
10	40	24	80%	2nd turn end		20	60%
5	45	28	80%	end		24	80%



The application of the prior to the level-off, 1st turn's 3.3 distance displacement to the turns after the level off will generate a conservative distance displacement margin to the turns after the level-off. In the above scale sample sketch, the conservative margin = 0.8 distance units (3.3 - 2.5).

If the climb gradient after the level-off is less than the climb gradient before the level-off; then, by geometry, the distance displacement after the level-off will result of a higher value than the distance displacement before the level-off.

Climb Gradient BEFORE LvLoff = 60%				Turnning climb gradient loss = 20%		
Climb Gradient AFTER LvLoff = 40%						
		NO_turn path			YES_turns path	
Δx	x	y	%		y	%
	0	0		start	0	
5	5	3	60%	1st turn start	3	60%
10	15	9	60%	1st turn end	7	40%
5	20	12	60%	LvLoff start	10	60%
5	25	12	0%	LvLoff end	10	0%
5	30	14	40%	2nd turn start	12	40%
10	40	18	40%	2nd turn end	14	20%
5	45	20	40%	end	16	40%



The application of the prior to the level-off, 1st turn's 3.3 distance displacement to the turns after the level off will generate a hampering error to the regulatory compliance displacement of the turns after the level-off. In the above scale sample sketch the hampering error = 1.7 distance units (5.0 – 3.3).

In the above 3 examples cases, note that the <u>height</u> difference value between NO and YES turn paths, always remains the same before and after the level-off. This attests that this only affects distance adjustment uncurl; it doesn't affect height adjustment uncurl.

As already stated; this distance inaccuracies result as a consequence of the level-off being uncurled with height. And it's now evident that the degree of inaccuracy will depend on the gradient after the level-off versus the gradient before the level-off.

Furthermore, if the gradients before and after the level-off are known, the ratio of these will serve to correct the inaccuracy. This can be done by taking the accumulated distance displacement just prior to the level-off and multiplying it by a correction factor resulting for the [climb gradient before the level-off / climb gradient after the level-off]. The result of this will be the accurate initial distance displacement that needs to be accounted to the turns that follow the level-off.

Examples:

When Climb Gradient BEFORE LvLoff = 60%
Climb Gradient AFTER LvLoff =80%

Correction factor = [60% / 80%] = 0.75

Distance displacement BEFORE the level-off = 3.3 distance units
Distance displacement AFTER the level-off with correction= 3.3 X 0.75 = 2.5 distance units

(In the prior scale sample sketch is was measured that 2.5 is the accurate distance units.)

When Climb Gradient BEFORE LvLoff = 60%
Climb Gradient AFTER LvLoff =40%

Correction factor = [60% / 40%] = 1.5

Distance displacement BEFORE the level-off = 3.3 distance units
Distance displacement AFTER the level-off with correction= 3.3 X 1.5 = 5.0 distance units

(In the prior scale sample sketch is was measured that 5.0 is the accurate distance units.)

Although the use of the correction factor is a valid alternative, its application might present a cumbersome additional task. In addition to this, it's a fact that by far, the most common occurrence is for the climb gradient after the level-off to be of a higher value than the climb gradient before the level-off. For this common occurrences, the result would be a conservative value, rather than a hampering to regulatory compliance error. This being the case, the performance engineer might find it practical to simply assume the inaccuracy as an added conservatism that allows for simplicity if the end results still proves convenient and appropriate. If the results rather show performance penalties due to over-conservatism, then the correction could be implemented.

Application
> Uncurl of level-off

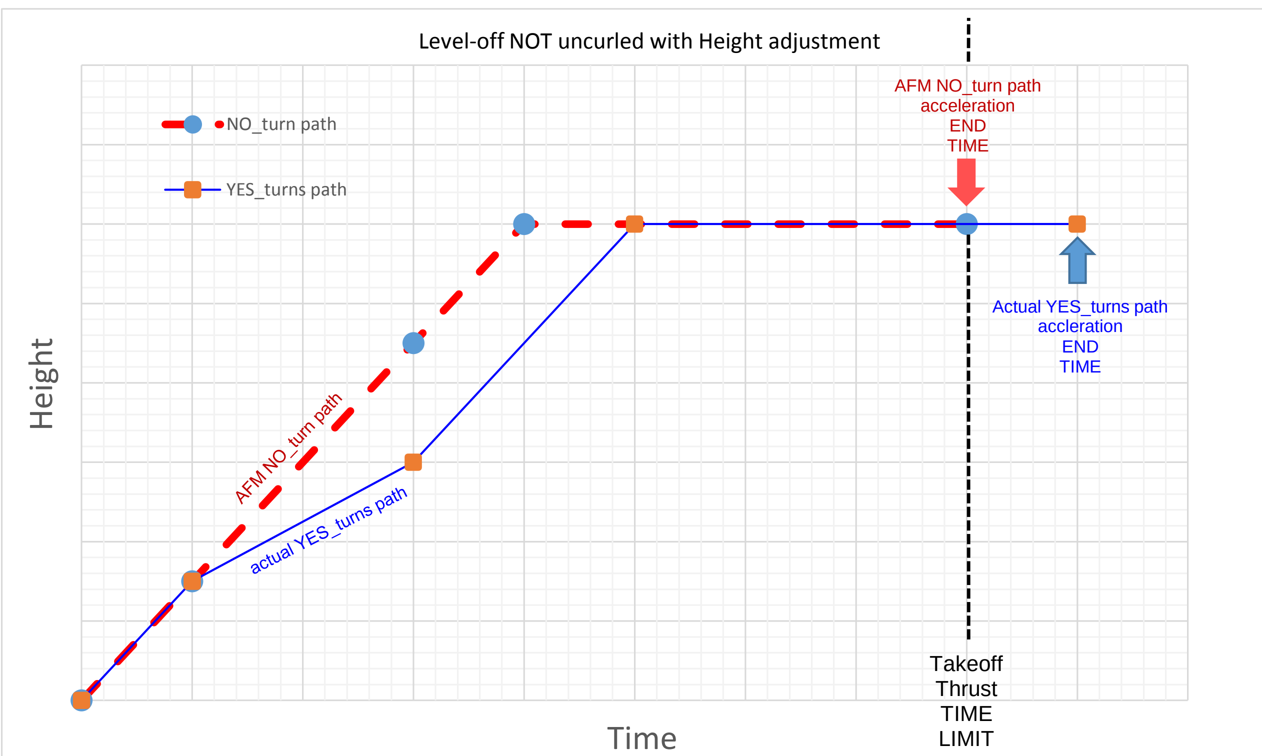
Regardless of the existence or not of obstacles; height adjustments applicable to the level-off, will result from the uncurl of turns before the level-off and from the uncurl of turns during the level-off.

The implementation of these level-off height adjustments will be in the form of a difference between the level-off information provided to the flight crew, and the information used by AFM/SCAP for performance calculations.

The need for these level-off height adjustments originate from the safeguarding of the Takeoff Thrust Time limits.

Application
> Uncurl of level-off
>> Turns <u>before</u> the level-off segment

With the existence of turns before the level-off; if the level-off information provided to the flight crew is the AFM NO_turn level-off height, then the actual aircraft's YES_turn path will reach the level-off at a later time than the AFM NO_turn path calculated.

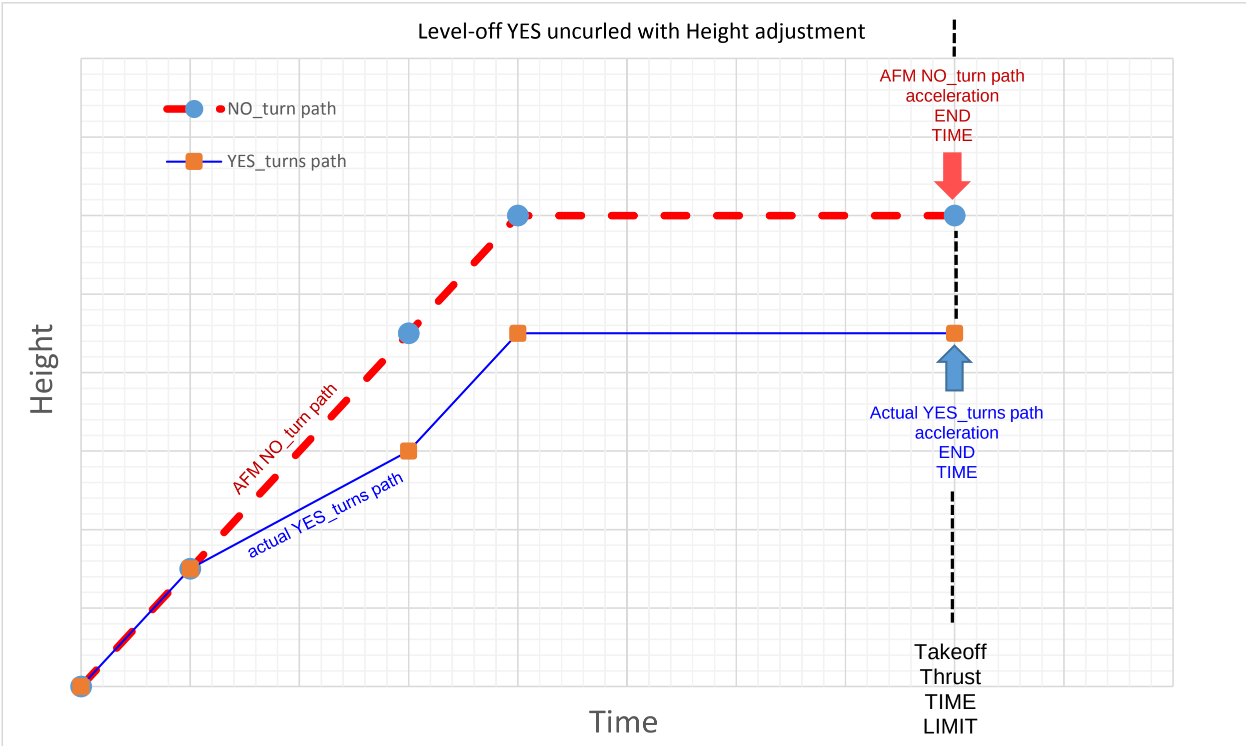


Note that in the above path plotting, the horizontal axis is "Time".
Keep in mind that at constant velocities, Time is proportional to Distance. In turn, at constant V_2 airspeed: Time will be reasonably proportional to Distance.

The Takeoff Thrust Time limit might be exceeded.
In the cases of 2nd Segment vertical profile and Final Segment vertical profile the exceedance will be at the end of the acceleration segment as the above sketch shows.

In the case of Extended Second Segment vertical profile, the exceedance will be when reaching the AFM NO_turn level-off height.

If the actual aircraft YES_turn path is made to level-off at the same Time that the AFM NO_turn path levels-off, then the Takeoff Thrust Time limit will result safeguarded to not be exceeded.



As noted earlier; recognize that the climb is performed at constant V2 airspeed. As a result, Time does hold a fairly reasonably constant correlation to Distance. In turn, with horizontal and vertical axis in distance units; a level-off height adjustment can be calculated by:

Height adjustment = Turn Track Distance X 2nd Segment GradDec

This will provide the desired result of the actual aircraft YES_turn path leveling-off at about the same Time that the AFM NO_turn path levels-off.

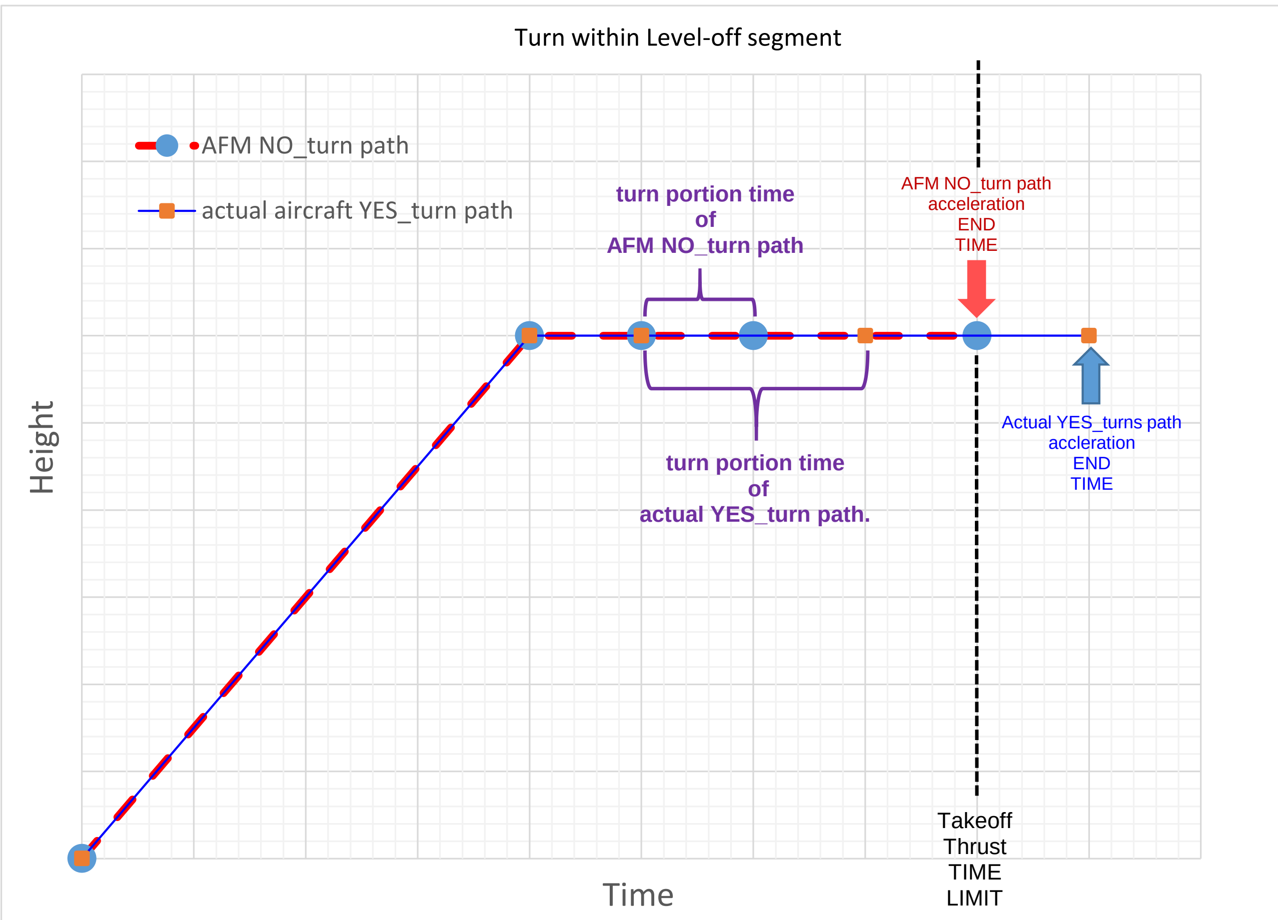
This will work for all vertical profiles: Second Segment, Final Segment, and Extended Second segment vertical profiles.

Application
> Uncurl of level-off
>> Turns <u>during</u> the level-off segment

This one only applies to 2nd Segment vertical profile and Final Segment vertical profile. It does not apply to Extended Second Segment profile, because the Extended Second Segment profile's acceleration is at Maximum Continuous Thrust and it does not have a Time limit.

This sub-section study of "Turns during the level-off segment"; is also a little more extensive to evaluate. After all; it's within an accelerated motion with varying configuration.

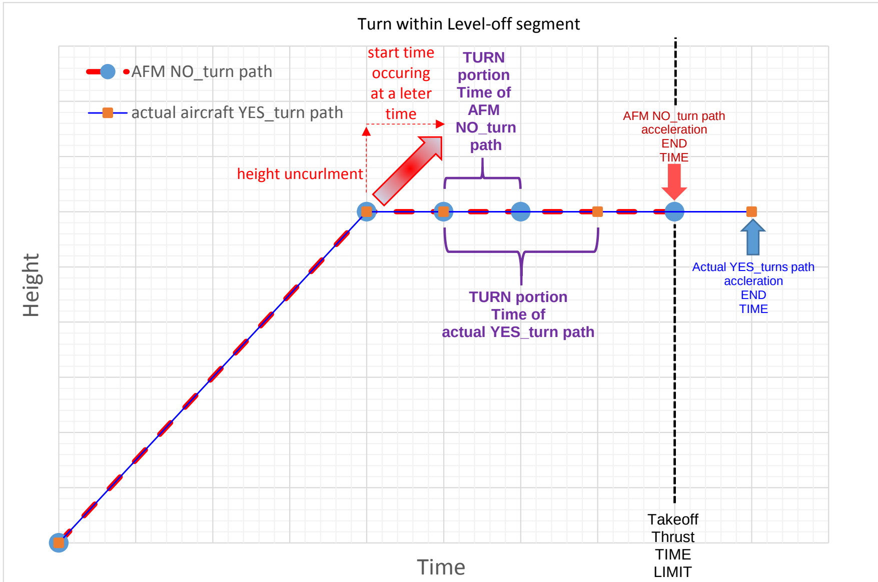
Acceleration capability decreases as a consequence of turning. Therefore, the YES_turn path will take a longer time than the AFM NO_turn path, to complete the level-off segment to a target END velocity. Because of this, the Takeoff Thrust time limit that the AFM NO_turn path protects, could be exceeded by the YES_turn path.



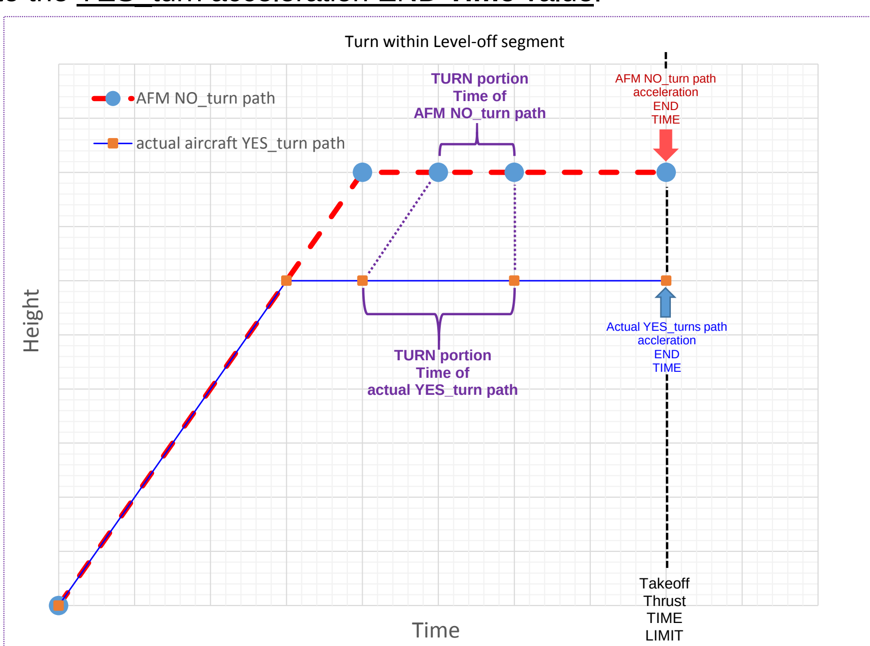
This problem can be addressed with a height uncurl adjustment.

Application
> Uncurl of level-off
>> Turns <u>during</u> the level-off segment
>>> Height uncurl adjustment for turns <u>during</u> the level-off segment

Initially, it could sound counterintuitive to increase the height of the AFM NO-turn path since that would seem to make acceleration start time to occur at a later time.



Yet, precisely because it's the AFM calculated path, it can be made to ensure the fulfillment of acceleration within the Takeoff thrust time. The sought after consequence of this, is that:
After a height uncurl of the level-off, the <u>AFM NO_turn path acceleration END **Time** value</u> will be greater than or equal to the <u>YES_turn acceleration END **Time** value</u>.



A height uncurl adjustment to the level-off that ensures this, can be calculated by:

$$\Delta\ height\ uncurl = Turn\ Track\ Distance\ \times 2nd\ Sgmnt\ GradDec\ due\ to\ turning$$

Again; to validate this method, it needs to be substantiated that:

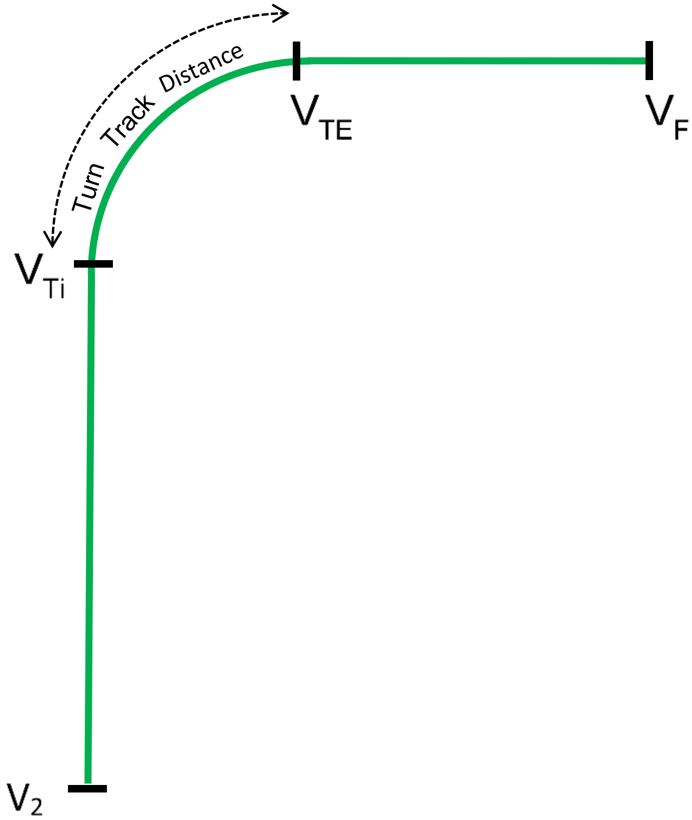
AFM NO_turn Accel_END Time ≥ YES_turn Accel_END Time

This substantiation is judged to be important; but at times it may seem counterintuitive or deceiving if not derived with care. Therefore it's judged best to start from the basics and work slowly step-by-step.

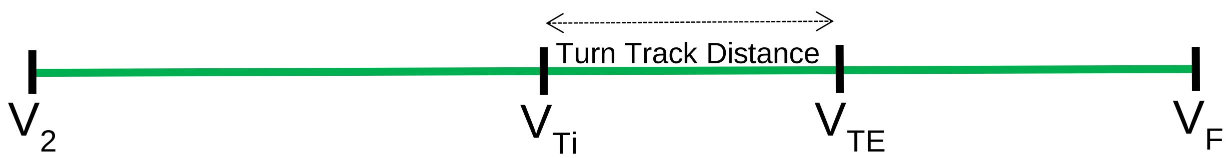
Consider a turning track schematic, during a level-off acceleration segment, as follows:

The aircraft start the level_off at V_2.
The aircraft first accelerates, wing level (no turn), until a Turn Start location in within the level_off.
At the Turn Start location, the velocity is V_{Ti} (Velocity at Turn initiation).
Turning Track Distance will be assumed to be a fixed distance.
The aircraft ends the Turn with velocity VT_E (Velocity at Turn End).
Then, the aircraft continues to accelerate, wings level (no turn), until a Final Velocity (V_F) is reached.

Top View.

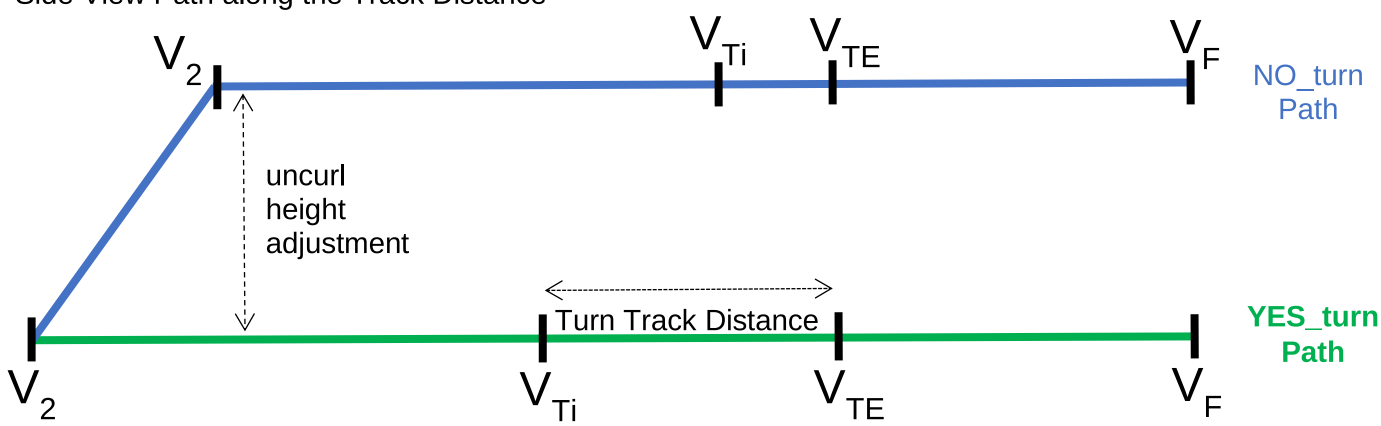


Side View Path along the Track Distance.



The turning YES_turn Path along the Track Distance is compared against a NO_turn path where an uncurl height adjustment is added just prior to the start of this NO_turn path's level-off.

Side View Path along the Track Distance



From aircraft performance fundamentals: Recognize that climb gradient capability, represents the aircraft's power available to either perform a rate of climb at constant airspeed, or accelerate at constant altitude, or any combination in between.

The acceleration on the YES_turn path, starts where the V_2 is indicated in the path.
The acceleration on the NO_turn path, starts immediately after the path has climbed the uncurl height adjustment at constant V_2.

The uncurl height adjustement is calculated as:

$$uncurl\ height\ adjustment = Turn\ Track\ Distance\ \times 2nd\ Sgmnt\ GradDec\ due\ to\ turning$$

The END Time of the YES_turn acceleration path occurs at V_F.
The END Time of the NO_turn acceleration path also occurs at V_F.

The case to substantiate is that:

The AFM NO_turn Accel_END <u>Time</u> ≥ The YES_turn Accel_END <u>Time</u>

The boundaries of the substantiation, will be set with two extreme cases:

Case 1: the 2nd Sgmnt Gradient Decrement due to Turning = the 2nd Sgmnt Climb Gradient capability.

This will entail that when turning, the aircraft is not capable of accelerating. Remember: climb gradient capability, represents the aircraft's power available.

Case 2: the 2nd Sgmnt Gradient Decrement due to Turning = 0.

This will entail that the YES_turn performance during acceleration will be identical to the NO_turn acceleration performance. They both have the same acceleration capability.
It also entails that calculated uncurl height adjustment will equal to zero.

It's worth mentioning that these two case boundaries; although they might seem unrealistic or improbable to say the least, in this substantiation they serve key for "encasing" the dynamics. Grasping a better mental view. And in turn, for facilitating the understanding of the substantiation. This author suggests to the reader to keep these boundaries in mind while working through the substantiation.

Case 1 boundary:

The 2nd Segment Gradient decrement due to Turning = the 2nd Segment Climb Gradient capability.

As a consequence; on the YES_turn path, when the aircraft turns, it will not accelerate. It will maintain the same velocity throughout the turn.

The NO_turn path's available acceleration is not affected.

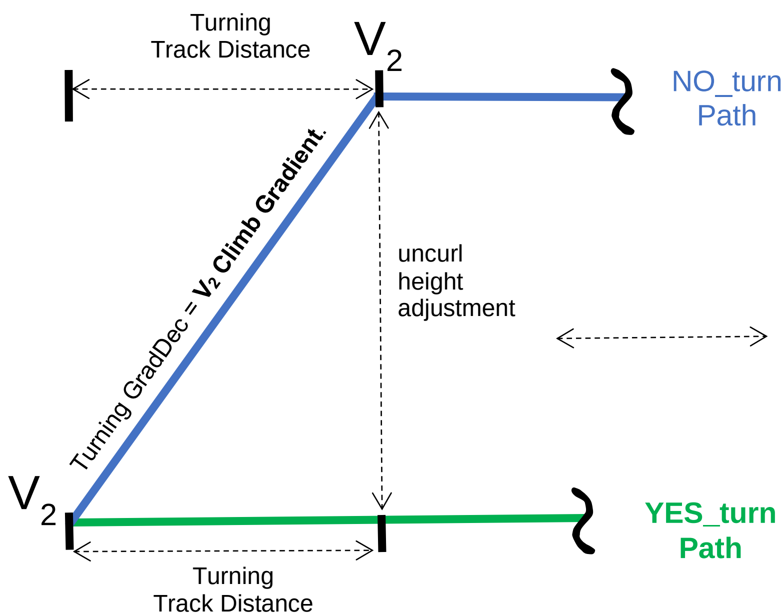
On the NO_turn path; the uncurl height adjustment is calculated with:

$$uncurl\ height\ adjustment = Turn\ Track\ Distance\ \times 2nd\ Sgmnt\ GradDec\ due\ to\ turning$$

Yet, because the $2nd\ Sgmnt\ GradDec\ due\ to\ turning$ is equal to the $2nd\ Sgmnt\ Climb\ Gradient$ for this particular boundary case, the uncurl height adjustment results the same as being calculated with:

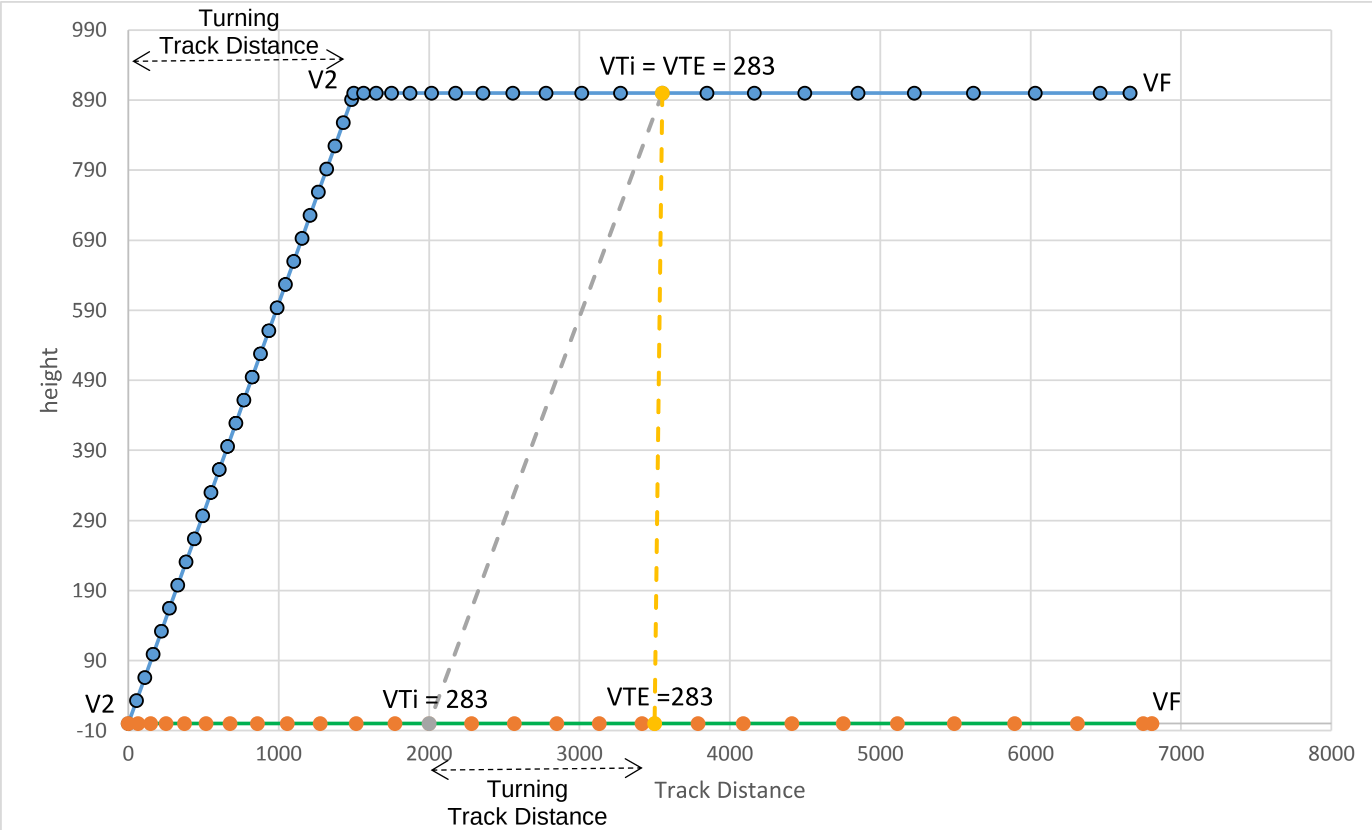
$$uncurl\ height\ adjustment = Turn\ Track\ Distance\ \times 2nd\ Sgmnt\ Climb\ Gradient$$

Since by definition, the $2nd\ Sgmnt\ Climb\ Gradient$ is the height gained per Track Distance traveled, the result of the above equation will be such that during the climbing of this boundary case calculated $uncurl\ height\ adjustment$, the track distance travelled will equal the Turning Track Distance.



The following will be assumed:
The degradation of acceleration capability due to the uncurl height difference between the NO_turn and the YES_turn path is assumed negligible. In other words, the acceleration capabilities of the NO_turn and the YES_turn path are assumed to be reasonably equal.

The plotting of an arbitrary made up test of this “case 1”, reveal certain details visible.



note: The above plot’s points, axis values and velocity numbers shown are from an arbitrary made up dynamic tests. They are for visual reference only. They do not abide to any given actual aircraft.

Recall: V2 = V_2
VTi = Velocity that results at the Start of the Turn on the YES_turn path.
VTE = Velocity that result at the END of the Turn on the YES_turn path.
VF = Target Velocity that defines the End of the acceleration segment for both the YES_turn and the NO_turn path.

In the segment between V2 and VTi points for both the YES_turn and the NO_turn paths, since both are accelerating at equally no turning conditions and both start from the same V2 velocity, <u>the Distance and Time increment needed to reach VTi will be the same</u> on both the NO_turn and YES_turn paths. For visualization of this, a dotted line is sketched between the VTi point on the NO_turn path and the VTi point on the YES_turn path.

For the YES_turn path, since [GradDec due to Turning = Climb Gradient capability]; during the turn, the aircraft will not be able to accelerate. Velocity will remain constant until the end of the “Turnning Track Distance” interval is completed. Therefore the velocity at the END of the Turn (VTE) will equal VTi.

Since VTi and VTE are defined from the YES_turn path and because in this case they are the same; on the NO_turn path, since the acceleration is continuous, the VTi and VTE value will result in a single track distance point for the NO_turn path.

In regard to VTE, for visualization, a dotted line is sketched between the VTE point on the NO_turn path and the VTE point on the YES_turn path.

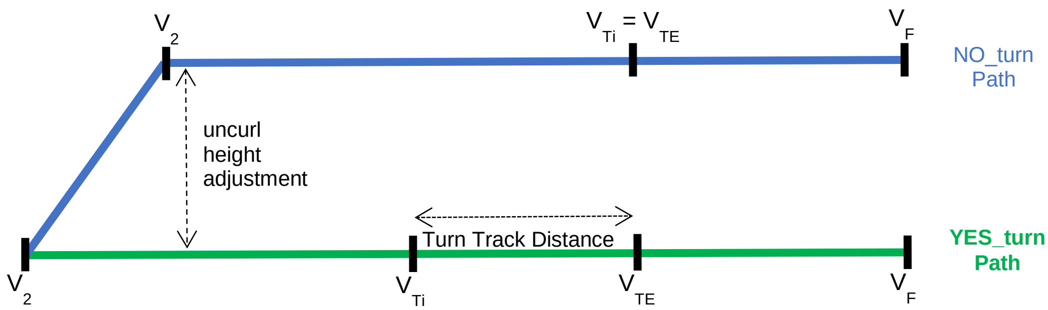
In the segment between the VTE and VF points for both the YES_turn and the NO_turn paths, since both are accelerating at equally no turning conditions and both start from the same VTE velocity; within the segment, <u>the Distance and Time increment needed to reach VF will be the same</u> on both the NO_turn and YES_turn paths.

The segments whose times remain to be compared is the turning track distance segment on the YES_turn path, against the climbing of the uncurled height at constant V_2 on the NO_turn path.

We already know that for this comparison, the Track Distances are the same and equal to the complete Turning Track Distance. Yet and furthermore, it's recognized that the average velocity during the turn on the YES_turn track will never be less than V_2.
($V_2 \leq VTi$) & ($VTi \leq$ average velocity during the turn $\leq VTE$)

Since the average velocity during the turn on the YES_turn path is always greater than or at least equal to the average velocity of the climbing of the uncurled height on the NO_turn path, and this segment's Track Distance is the same for both the YES_turn and the NO_turn paths; it will always take a longer or at least an equal time for the NO_turn path to travel this distance.

Compiling all the segments for the Case 1 boundary
(2nd Sgmnt Gradient decrement due to Turning = the 2nd Sgmnt Climb Gradient capability)



Within accelerating Segment from V_2-to-VTi : NO_turn Path Time = Yes_Turn Path Time
YES_turn VTi-to-VTE -vs- NO_turn Clb_at_V_2 : NO_turn Path Time ≥ Yes_Turn Path Time
Within accelerating Segment from VTE-to-VF : NO_turn Path Time = Yes_Turn Path Time

As a result, for this Case boundary the [NO_turn Path Time] will always be ≥ [Yes_Turn Path Time]. In turn, since the AFM protects Time limit on the NO_turn Path, the Time limit on the YES_turn Path will also result protected.

A conservatism expected trend behavior can be resolved from this special boundary case analysis. The Turning Track Distance comparison between the NO_turn path and the YES_turn path is the only differentiator between the two path's complete acceleration segment in this case analysis. And the pivotal difference lies in the difference in average velocity that exists between these segments of the two paths. The YES_path's average velocity during turning acceleration will always be greater than or equal to the NO_turn average velocity during constant V_2 climb. The larger the average velocity of the YES_turn path, the more conservative the NO_turn path will be. The further into the level-off acceleration segment, that the Turn Start point exists; the lager the VTi will be, and in turn, the larger the turning average velocity will result. In consequence the further away the Turn Start is from where the level-off acceleration segment initiates, the more conservative the NO_turn path will be. The closer that the Turn Start is to the where the level-off acceleration segment initiates, the less conservative the NO_turn path will be. Yet; in all cases, it will always be safe.

Case 2 boundary:

The 2nd Sgmnt Gradient decrement due to Turning = 0.

As a consequence; during the turn segment in the YES_turn path, its acceleration is the same as the NO_turn path's acceleration. Furthermore this makes the dynamics to be the same for the two paths. The Time and Distances at given speeds are identical in the YES_turn path and in the NO_turn path.

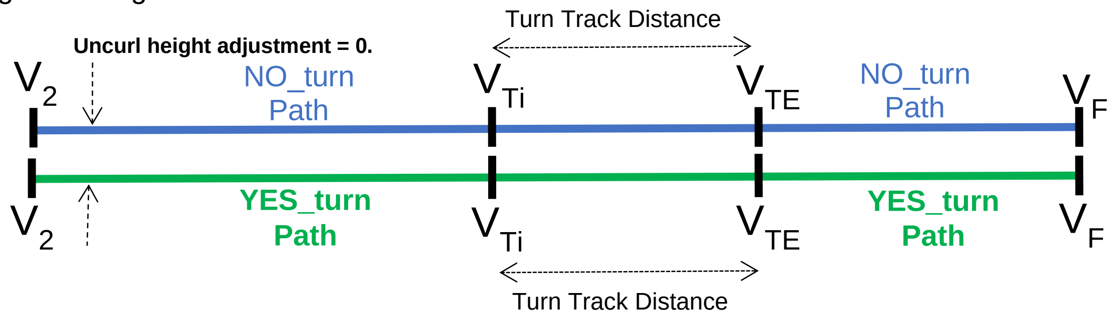
Another consequence, is that the $uncurl\ height\ adjustment$ will also equal to zero.

On the NO_turn path; the uncurl height adjustment is calculated with:

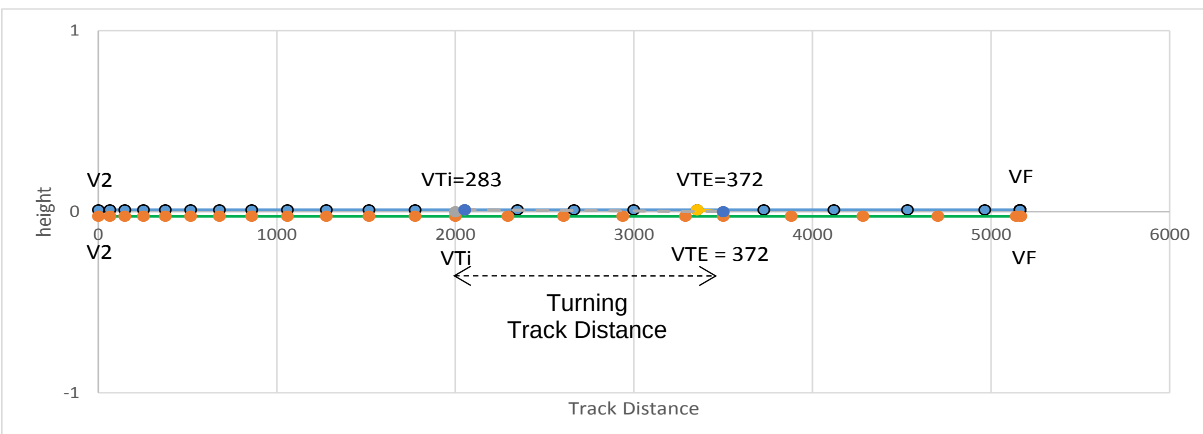
$$uncurl\ height\ adjustment = Turn\ Track\ Distance\ \times 2nd\ Sgmnt\ GradDec\ due\ to\ turning$$

Since [2nd Sgmnt Gradient decrement due to Turning = 0], then $uncurl\ height\ adjustment = 0$.

Sketching this along the Track Distance:



This can also be plotted from an arbitrary made up test of this "case 2" (this plot will soon serve useful).



note: The above plot's points, axis values, and velocity numbers shown are from an arbitrary made up dynamic tests. They are for visual reference only. They do not abide to any given actual aircraft.

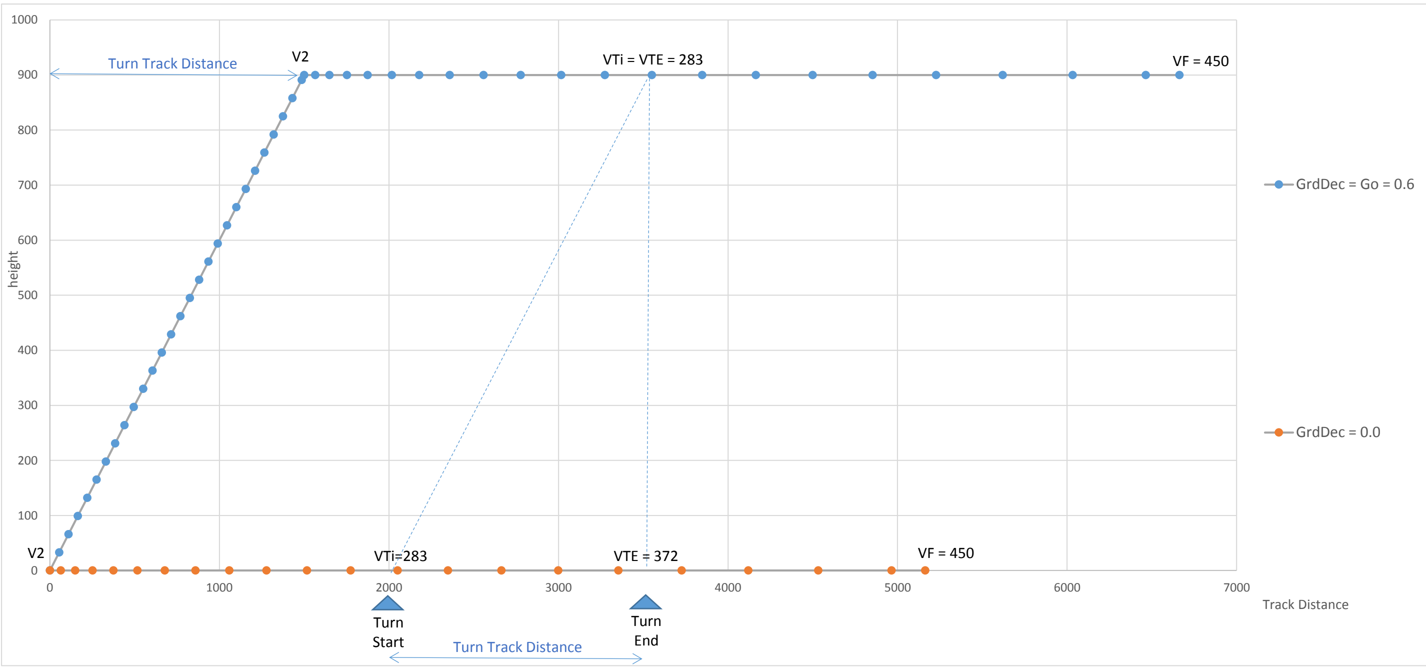
Compiling all the segments for the Case 2 boundary

Within accelerating Segment from V_2-to-VTi :	NO_turn Path Time = Yes_Turn Path Time
Within accelerating Segment from VTi-to-VTE :	NO_turn Path Time = Yes_Turn Path Time
Within accelerating Segment from VTE-to-VF :	NO_turn Path Time = Yes_Turn Path Time

As a result, for this Case boundary the [NO_turn Path Time] = the [Yes_Turn Path Time].
In turn, since the AFM protects Time limit on the NO_turn Path, the Time limit on the YES_turn Path will also result protected.

Now, it serves convenient to plot both NO_turn paths of the two Boundary Cases in one single plot.

Boundary Case 1: the 2nd Sgmnt Gradient decrement due to Turning = the 2nd Sgmnt Climb Gradient capability.
Boundary Case 2: the 2nd Sgmnt Gradient decrement due to Turning = 0.



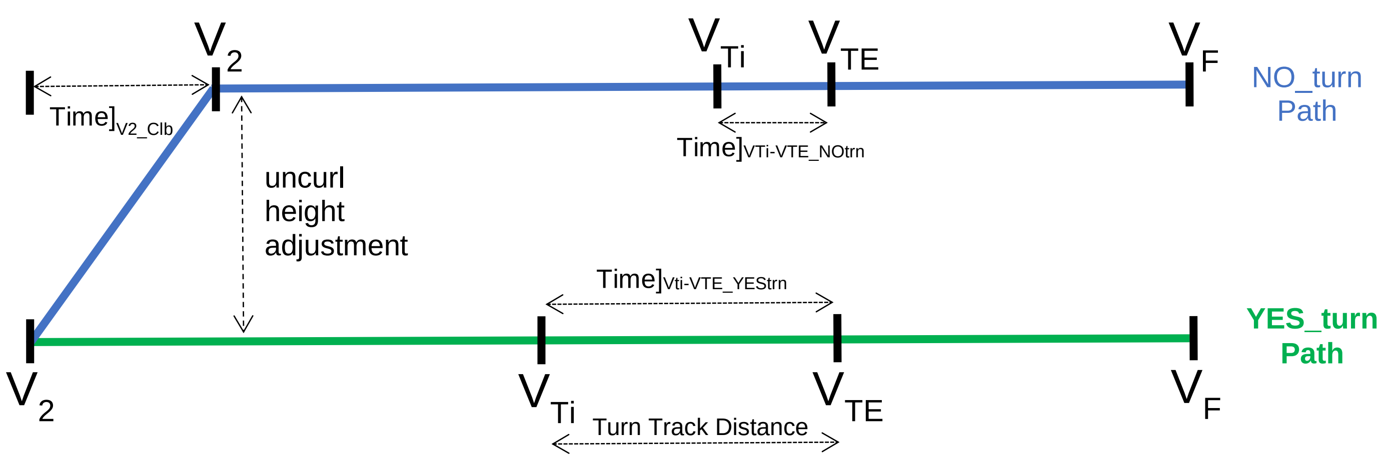
note: The above plot's points, axis values and velocity numbers shown are from an arbitrary made up dynamic tests.
They are for visual reference only. They do not abide to any given actual aircraft.

The label "G_o" is used to represent the 2nd Segment Climb Gradient capability.

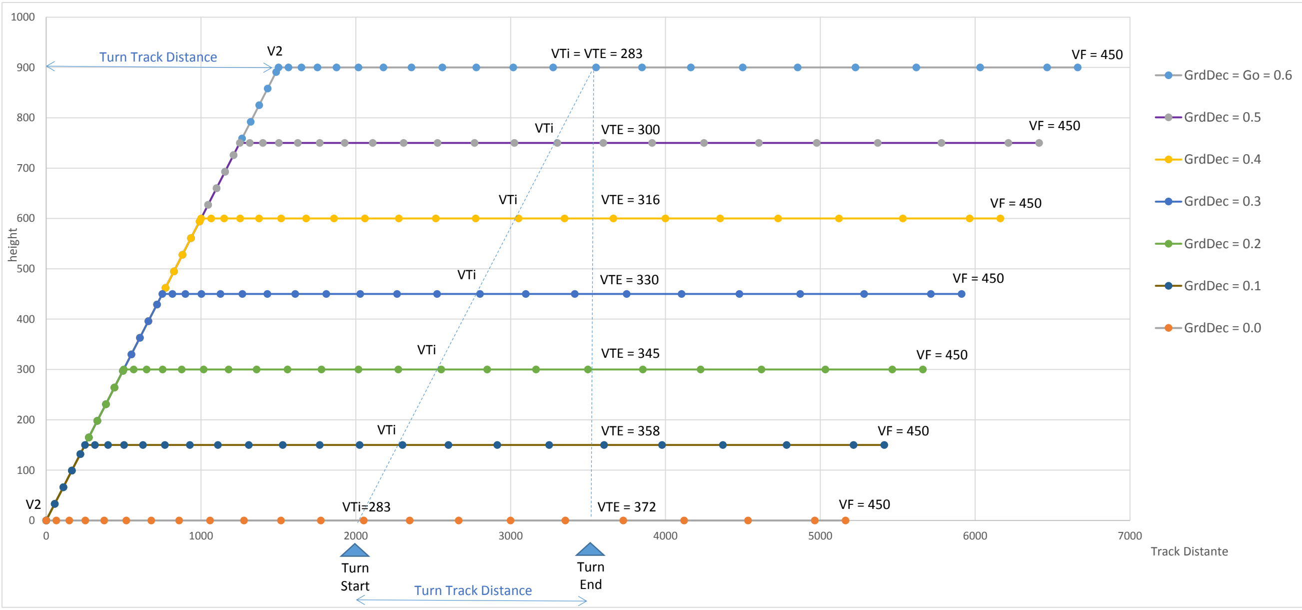
It has already been proven that at both boundaries cases, [NO_turn Path Time] ≥ [Yes_Turn Path Time]. So now, as Gradient Decrement varies from [GradDec=G_o] toward [GradDec=0], the behavior of the conditions when VTi and VTE exist become of interest. For this purpose, note that in the above plot, the conditions where VTi and VTE exist in both boundary cases are joined with light dotted lines. Although VTE have different values in the [GradDec=G_o] and the [GradDec=0] boundary cases.

The segments before and after any given VTi and VTE, the aircraft is not turning. Therefore the dynamics of Time and Distances at given speeds will be the same in the NO_turn as in the YES_turn paths. For any particular GradDec, it will be the differences in Time that exist in the sections between VTi and VTE for each corresponding NO_turn and YES_turn path, that the extra Track Distance Time of the constant V2 climb on the NO_turn path must compensate.

$$\text{Time}]_{V2\_Clb} \geq \left(\text{Time}]_{VTi\text{-}VTE\_YEStrn} - \text{Time}]_{VTi\text{-}VTE\_NOtrn} \right)$$



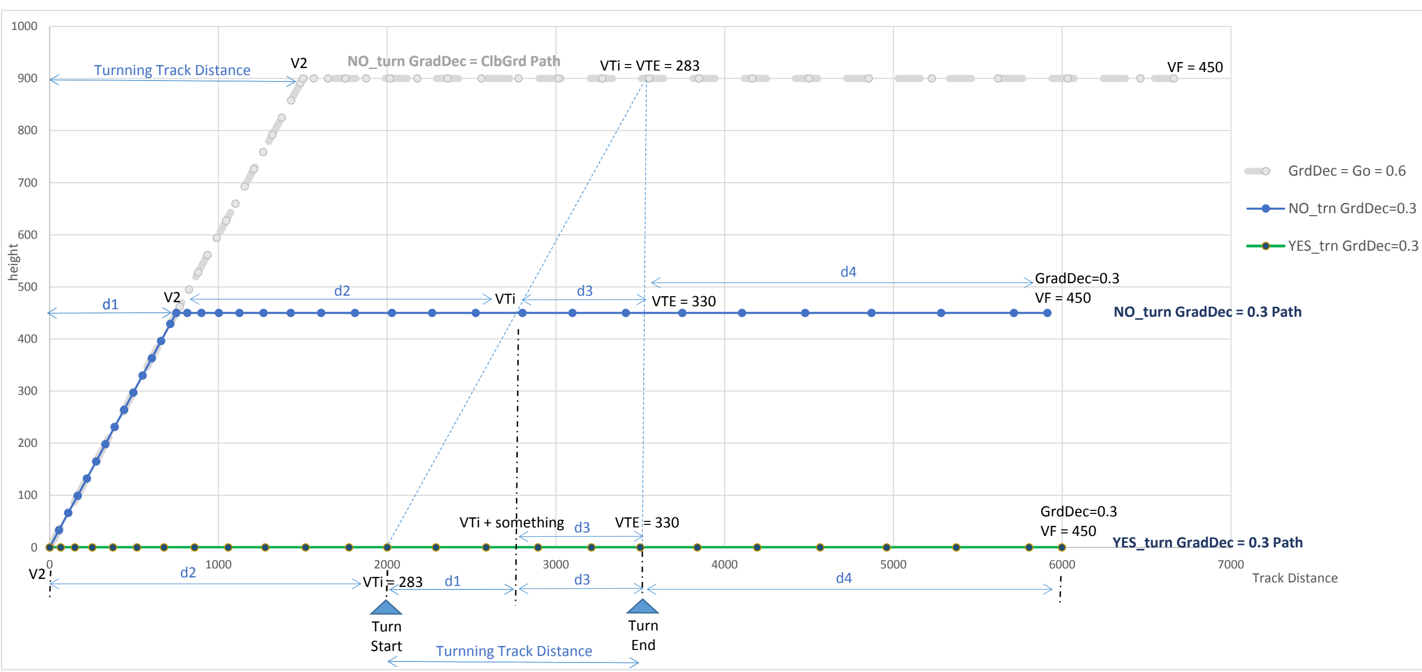
Intuitively, one would expect and hope that the resolution of VTi and VTE does result along the dotted lines in the prior presented plot. To confirm the actual behavior, arbitrary made up dymanic test were done for Gradient Decrements between [GradDec=G_o] toward [GradDec=0]. These demonstrated the intuition and hope as correct.



note: The above plot's points, axis values and velocity numbers shown are from an arbitrary made up dynamic tests. They are for visual reference only. They do not abide to any given actual aircraft.

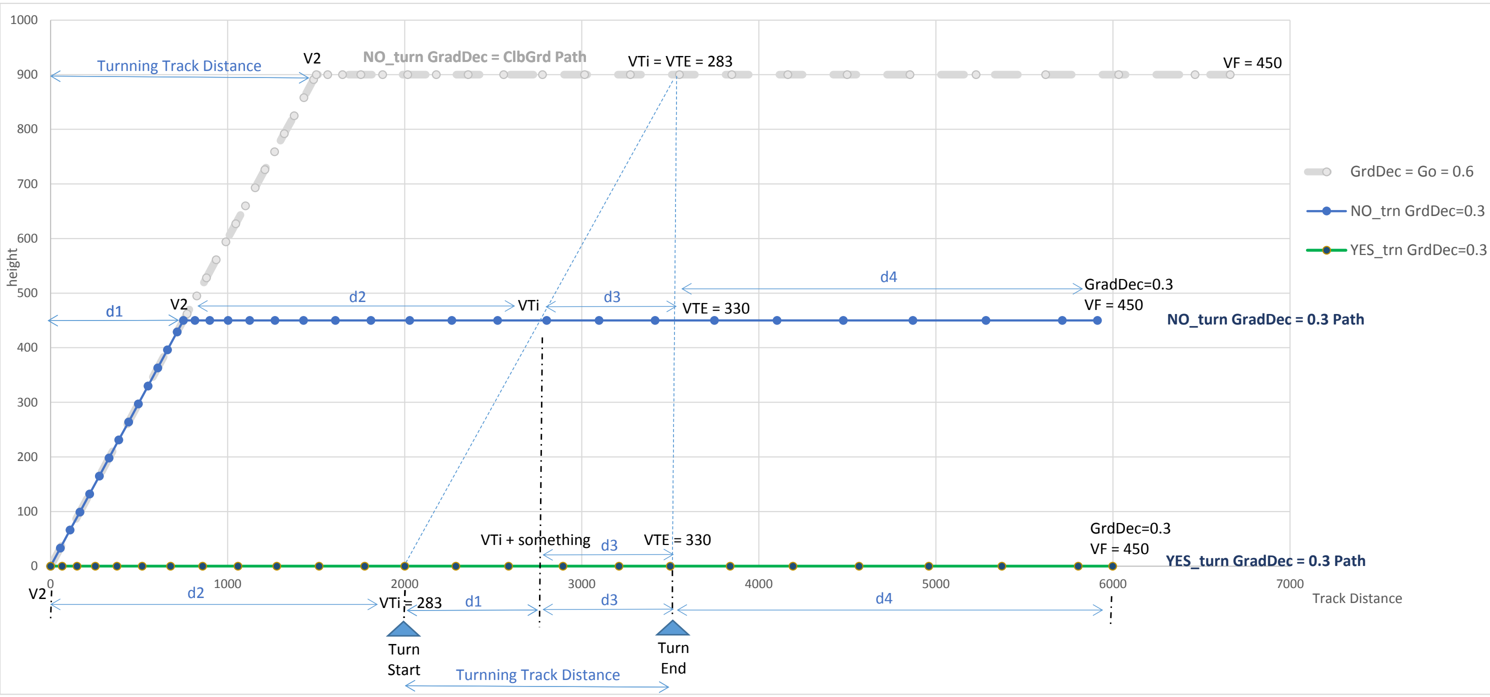
Remember, the above plot are only the plotting of the NO_turn paths.

Next, one of the tests between the boundaries, is arbitrarily taken for further examination. GradDec=0.3 is chosen. And to this, we will now plot both the NO_turn path and the YES_turn path. Also similar segments on the NO_turn and YES_turn paths will be labeled as "d1", "d2", "d3", and "d4". *Also note, the NO_turn Boundary Case of [GradDec = G_o] is shown with a light color dashed line as reference.*



The hopes have been that just as the Boundary cases proved compliance with
[NO_turn Path Time] ≥ [Yes_Turn Path Time],
that the in between boundary cases will as prove compliant.

With the "d1" through "d4" segments defined, the compliance of [NO_turn Path Time] ≥ [Yes_Turn Path Time] can be made through logic arguments. *The prior plot is re-shown for convenience of the reader.*



d1

By geometry, the Track Distance is the same for the NO_turn "d1" and the YES_turn "d1".
But the NO_turn "d1" resolves at a lower average velocity than the YES_turn "d1".
Therefore; the NO_turn "d1" at lower average velocity consumes longer Time.
Therefore; the NO_turn "d1" TIME > YES_turn "d1" TIME.

d2

By geometry, the Track Distance is the same for the NO_turn "d2" and the YES_turn "d2".
The initial and end velocities are also the same for the NO_turn "d2" and the YES_turn "d2".
Since no turning is occurring in both NO_turn "d2" and the YES_turn "d2", the acceleration is the same.
Therefore; the dynamics of TIME and Distances at given speeds will be the same.
Therefore; the NO_turn "d2" TIME = YES_turn "d2" TIME.

d3

By geometry, the Track Distance is reasonably the same for the NO_turn "d3" and the YES_turn "d3".
And the END velocity of the NO_trun "d3" and YES_Turn "d3" are the same. *(confirmed in tests)*
But, the YES_turn "d3" has higher initial velocity. *(VTi + something)*
Therefore; YES_turn "d3" has higher average velocity.
Therefore; NO_turn "d3" has lower average velocity.
Therefore; the NO_turn "d3" at lower average velocity consumes longer Time.
Therefore; the NO_turn "d3" TIME > YES_turn "d3" TIME.

d4

By geometry, the Track Distance is the same for the NO_turn "d4" and the YES_turn "d4".
The initial and end velocities are also the same for the NO_turn "d4" and the YES_turn "d4".
Since no turning is occurring in both NO_turn "d4" and the YES_turn "d4", the acceleration is the same.
Therefore; dynamics of TIME are the same; hence, the NO_turn "d4" TIME = YES_turn "d4" TIME.

Compiling segments “d1” through “d4” for the any particular case between the Boundary Cases:

Within “d1” segment: NO_turn Path Time > Yes_Turn Path Time

Within “d2” segment: NO_turn Path Time = Yes_Turn Path Time

Within “d3” segment: NO_turn Path Time > Yes_Turn Path Time

Within “d4” segment: NO_turn Path Time = Yes_Turn Path Time

As a result; on this sub-section “**Height uncurl adjustment for turns <u>during</u> the level-off segment**”, for the boundary cases and any in between case:

[NO_turn Path Time] ≥ [Yes_Turn Path Time].

In turn, since the AFM protects Time limit on the NO_turn Path, the Time limit on the YES_turn Path will also be protected by an uncurl height adjustment calculated with:

$$uncurl\ height\ adjustment = Turn\ Track\ Distance\ \times 2nd\ Sgmnt\ GradDec\ due\ to\ turning$$

Application
> Uncurl of level-off
>> Turns <u>during</u> the level-off segment
>>> Distance uncurl adjustment for turns <u>during</u> the level-off segment

Taking the height uncurl adjustment as a basis; a Distance uncurl adjustment can be formulated by dividing the uncurl height adjustment by the 2nd Segment Climb Gradient just prior to the level off.

$$uncurl\ distance\ adjustment = \frac{uncurl\ height\ adjustment}{2nd\ Sgmnt\ Climb\ Gradient}$$

Based on the validity of the height uncurl substantiation; this $uncurl\ distance\ adjustment$ will represent a distance which when travelled in level flight on the NO_turn path <u>at constant V_2</u>, it will provide an appropriate time that accounts for the additional “Time” that the YES_turn path consumes while turning due to the decrease acceleration that results from turning.

Nevertheless, although this uncurl distance adjustment can be calculated, with known AFM/SCAP technologies, it cannot currently be implemented to uncurl of “the level-off”. The implementation would require for the AFM to have the capability of incorporating a constant V_2 sub-segment in the level-off portion of its path, prior to the acceleration starting in the level-off segment.

Another perhaps simpler alternative to implement this, would be for an enhanced AFM/SCAP to be able to accept as an input, a reduction to the limit value of Takeoff Thrust Time allowed. For an AFM/SCAP possessing this capability; an uncurl TIME adjustment could be calculated by taking the prior calculated distance uncurl adjustment and dividing it by V_2.

$$uncurl\ TIME\ adjustment = \frac{uncurl\ distance\ adjustment}{2nd\ Sgmnt\ \ V_2}$$

Ensuring appropriate units conversions, the resulting uncurl TIME adjustment could be entered as an input to an AFM/SCAP, to in turn have it reduce the Takeoff Thrust Time limit value.

Regardless of application to level-off difficulties; uncurl distance adjustments could serve useful for further adjustments after the level-off.

Application
> Uncurl of level-off
>> Implementation of level-off height adjustment uncurl due to turns

For all level-off height adjustments due to turns that exist before and/or during level-off:

> The YES_turn path level-off values should be used as the basis for the level-off information to be provided to the flight crew.
>
> The NO_turn path level-off values should be used for the AFM and SCAP input variables for calculations.

If the level-off is bounded by a bottom limit height value, for example an obstacle, then this would represent the YES_turn path, and the uncurled higher height would be applied to the AFM NO_turn path.

If the level-off is bounded by a top limit height value, for example the most conservative maximum level-off condition, then this would represent the AFM NO_turn path. And the YES_turn path would be uncurled to lower heights (flight crew level-off information).

A probably not common but interesting case to consider and be aware of; would be one where at a given set of conditions, configuration, and performance options, the AFM results are bound by a top limit height value, and a bottom limit height value also exists, and the uncurl height adjustment from a turn, exceeds the height difference between the level-off's low and high height limit bounds. In this case, an AFM result will exist. But, the AFM NO_turn path would not be appropriately representing the actual aircraft's YES_turn path lower bounded limits. The performance engineer should notice this when attempting to obtain the level-off height information to convey to the flight crew. This situation must be corrected. Something in the conditions, configuration, or performance options should be changed.

Continuing with the above paragraph's case scenario; the performance engineer could ensure that the needed turn uncurl height adjustment prevails to exist, by raising the level-off upper bounds and setting the AFM's level-off inputs to a fixed level-off height equal to this adjusted upper bound. By doing this, the AFM might provide lower takeoff weight results when limited at the upper bounds; but always with the needed turn uncurl height adjustment.

Another perhaps more convenient way for the performance engineer to address this problem, could be by limiting some other configuration parameter to the point of ensuring that the needed turn uncurl height adjustment does fit between the level-off's upper and lower set bounds. For example: limiting flap setting, or bleed configuration. This could possibly permit the limit takeoff weights to remain high.

Remember, that the level-off information provided to the crew should originate from the YES_turn path. For the preceding three paragraphs case, this will more closely match the level-off height's lower bounds.

Engineering judgment must always prevail on the use and execution of all these processes.

For Extended Second Segment, differentiating between the level-off information provided to the flight crew and the level off information used for AFM/SCAP performance calculations, is the way in which the Takeoff Thrust Time limit is guaranteed not to be exceeded when turns exist during the Extended Second Segment. Since at Extended Second Segment vertical profiles, the AFM NO_turn level-off height will vary; it serves conservative for the level-off provided to the crew, to be an appropriate low one. Perhaps originating from an array of predicted possible case scenarios. Again, prudent engineering judgement is the best advice.

Application
> Uncurl of obstacles on 2nd segment

Either height or distance adjustment turn uncurl can be applied to obstacles in the 2nd segment. It's recognized that distance uncurl entails a little more work than height uncurl.

Given that the level-off is uncurled with height adjustment from any turn occurring during the 2nd segment; it serves convenient to uncurl 2nd segment obstacles with height adjustments, because of its simpler process. Especially close-in obstacles. By definition, obstacles' uncurl height increase would not be able surpass a height uncurled AFM NO_turn level-off.

It could happen that a turn only partially exist in the 2nd segment, with the remaining of the turn occurring during the acceleration segment. For this, the appropriate accounting solution would be to anticipate the maximum amount of the turn that could result during the 2nd segment and resolve the uncurl height adjustment only for this portion of the turn.

Engineering judgment must prevail on the use and execution of the above processes.

Application
> Uncurl of obstacles on Level-off segment

2nd segment (including Extended 2nd Segment) complete turn uncurl adjustments must be applied to obstacle that exists in the level-off segment. These 2nd segment uncurl adjustments can be either height or distance adjustments. Nevertheless, height uncurl adjustments are simpler to compute.

For turns that occur during the level-off; the obstacles that follow these turn's start location and are obstacles that exist inside the level-off; these obstacles can, but do not necessarily need to be uncurled in any way. Neither height nor distance uncurl.

For all obstacles within the level-off segment; the performance engineer must ensure that these obstacles are considered for setting the level-off bottom limit height value bounds applicable to the YES_turn flight path as described in the section "Implementation of level-off height adjustment uncurl due to turns". The value is accounted as the actual height of the tallest obstacles existing within the level-off, plus the 35ft, plus the biggest possible Net-to-Gross margin up the start of the level-off segment. Again, this must be accounted for setting the level-off lower limit bounds of the YES_turn flight path as described in the section "Implementation of level-off height adjustment uncurl due to turns".

Engineering judgment must prevail on the use and execution of the above processes.

Application
> Uncurl of obstacles on Final Segment

Complete turn uncurl adjustments from all 2nd segment turns and/or from all level-off segment turns, must be applied to obstacles that exists in the Final Segment. The 2nd segment and level-off uncurl adjustments can be either height or distance adjustments.

In the case of the level-off turns, recall that distance uncurl can be calculated, even if they were not implemented into the level-off. To this end, the level-off uncurl distance adjustment could be used as the adjustment to be accounted to the obstacles on Final Segment in lieu of level-off turn height adjustment to the obstacles.

Application
> Uncurl of obstacles on Final Segment (continuation)

For turns occurring during the Final Segment, either height or distance adjustment uncurl can be applied to obstacles in the Final Segment. It's recognized that distance adjustments entail a little more work than height adjustments.

Obstacles could exist in the Final Segment; that follow a turn that initiated during the level-off segment and extends into the Final Segment. For such a case, a turn could only partially exist in the Final Segment, with the initial part of the turn occurring during the level-off acceleration segment. In this case an uncurl adjustment for the part of the turn that corresponds to the level-off should be calculated based on level-off's parameters. And a separate uncurl adjustment for the part of the turn that corresponds to the Final Segment should be calculated based on Final Segment's parameters. Then both uncurl adjustments should be independently accounted to the obstacles on Final Segment. Furthermore, the type of these two uncurl adjustments (height or distance adjustments) do not necessarily need to be the same. They can still be any combination of height or distance adjustments.

At the final segment climb portion of a Final Segment vertical profile, the Gross-to-Net margin can be big. Especially towards the end. The AFM NO_turn Gross path will already be at a high height at this stage. If several turns exist, the accumulation of the turn's height adjustments could be significant. And when added with the Gross-to-Net margin, it will increase the AFM's NO_turn Gross height even further. Very high AFM Gross heights could bring limitation problems when evaluating the end of the Flight Path. Even particular engine manufacturer's specific data prediction availability problems might result. Uncurl height adjustments might end up imposing AFM higher Gross heights that are more hampering than expected. Meanwhile; the actual aircraft is not truly expected to be at those AFM Gross heights at the corresponding track distance and time instances. So; for far away obstacles, uncurl distance adjustments might be preferable for appropriately accounting turn uncurl effects on obstacles while also granting lower AFM Gross heights, so as to avoid unexpected additional problems.

Engineering judgment must prevail on the use and execution of the above processes.

Application
> Extended 2nd Segment obstacle uncurl

Within the constant V2 climb portion of the Extended 2nd Segment the uncurl application is very similar to the "standard" 2nd segment vertical profile. Either height or distance adjustment turn uncurl can be applied to obstacles in the extended 2nd segment. It's recognized that distance uncurl entails a little more work than height uncurl.

It could happen that a turn only partially exists in the constant V2 climb portion of the Extended 2nd Segment, with the remaining of the turn occurring during the following level flight acceleration segment at Maximum Continuous Thrust (MCT). For this, the appropriate accounting solution would be to anticipate the maximum amount of the turn that results during the constant V2 climb portion of the Extended 2nd Segment and resolve the uncurl height adjustment only for this portion the turn.

At the end of the constant V2 climb portion of the Extended 2nd Segment, the Gross-to-Net margin can be big. The AFM NO_turn Gross path will be at a high height. If several turns exist, the added turn's height adjustments will increase the height further. Very high AFM Gross heights could bring limitation problems: The Extended 2nd Segment MCT acceleration climb gradient requirement (1.2% for two engine aircraft) might cause weight limitations at higher heights. Clearance of height uncurled obstacles after level-off. Even particular engine manufacturer's specific data prediction availability problems might result. So again; for far away obstacles, distance adjustment uncurl of obstacles might be preferable.

Engineering judgment must prevail on the use and execution of the above processes.

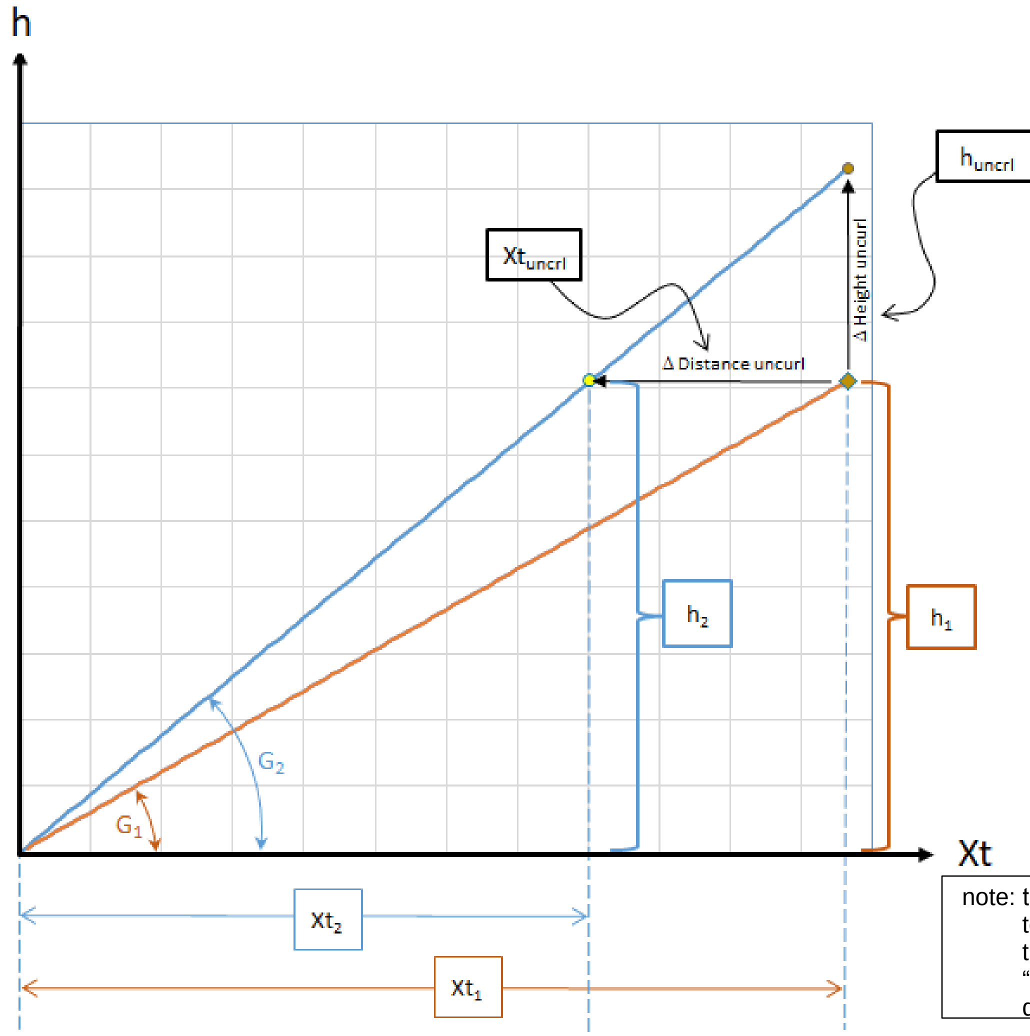
Application
> Application Conclusion synopsis

- Reliable Height and Distance adjustments for turn uncurl can be computed for all segments.
- Calculation of Height adjustment requires one less variable than Distance adjustment calculation.
- For Close-in obstacles, Height adjustment uncurl is convenient because it's simpler to calculate.
- For Far-away obstacles, Distance adjustment uncurl could provide convenient results.
- Level-off height adjustments due to turn uncurl, are independent of obstacle adjustments.
- Level-off height adjustments due to turn uncurl, are implemented by having the level-off height information provided to the flight crew being different from the level-off used for AFM/SCAP computations. Obstacles within the level-off must be accounted for setting the lower bound limits.
- Prudent Engineering Judgement is always of high value in the choosing between Height or Distance uncurl of turns and its application.

END

This paper is not the absolute truth; just a version.

Annex 1 - Height and Distance adjustment uncurl due to turns



note: the "t" in the "Xt" terms, stands for the axis being "turning" track distances

$$G = \text{gradient}$$

$$G = \frac{h}{Xt}$$

$$G\,Xt = h$$

premise: $h_2 = h_1$

$$G_2\,Xt_2 = G_1\,Xt_1$$

$$Xt_2 = \frac{G_1}{G_2}Xt_1 \quad \text{(Eq1)}$$

premise: $Xt_{uncrl} = Xt_1 - Xt_2$ (Eq2)

substituting (Eq1) into (Eq2) :

$$Xt_{uncrl} = Xt_1 - \frac{G_1}{G_2}Xt_1$$

$$Xt_{uncrl} = Xt_1\left(1 - \frac{G_1}{G_2}\right)$$

Remember,… G = gradient ; and since $G_1 = G_2 - \text{GrdDec}$,..

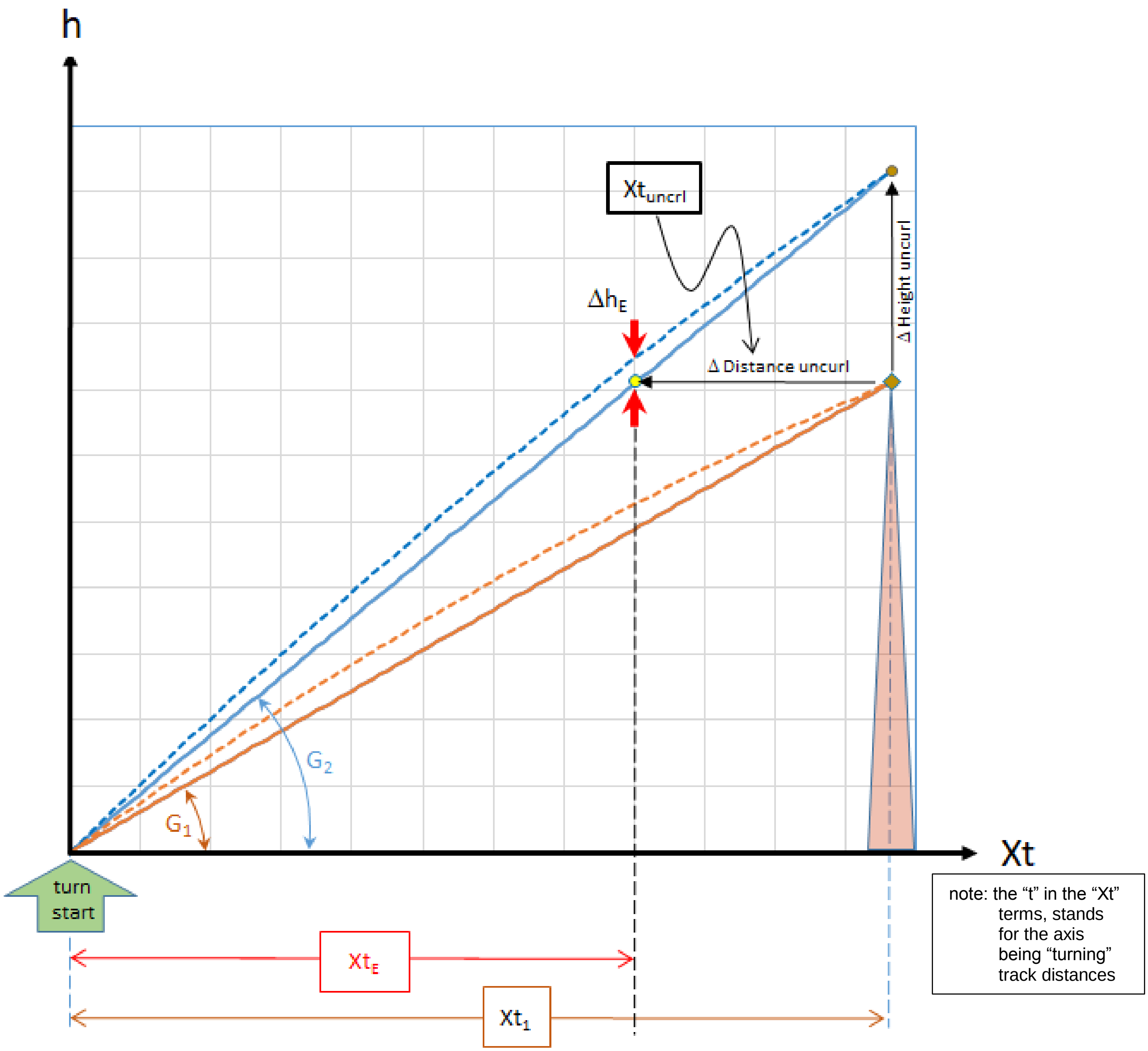
$$Xt_{uncrl} = Xt_1\left(1 - \frac{G_2 - \text{GrdDec}}{G_2}\right)$$

$$Xt_{uncrl} = Xt_1\left(1 - \frac{G_2}{G_2} + \frac{\text{GrdDec}}{G_2}\right)$$

$$Xt_{uncrl} = Xt_1\left(1 - 1 + \frac{\text{GrdDec}}{G_2}\right)$$

$$\boxed{Xt_{uncrl} = Xt_1\left(\frac{\text{GrdDec}}{G_2}\right)}$$

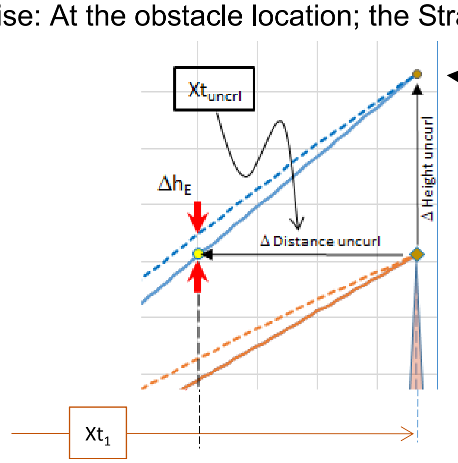
Annex 1 - Height and Distance adjustment uncurl due to turns



Premise: The Actual Flight Path is to be modeled as a parabola with Equation: $\mathrm{A}\,(\mathrm{Xt})^2 + \mathrm{B}\,\mathrm{Xt} + 0 = \mathrm{h}$

(where "A" and "B" are the constants)

Premise: At the obstacle location; the Straight Lines and the Actual Flight Paths have the same "h" value:



Same "h". The straight line and the curved FltPath, intersect.

$$\mathrm{h}\ \ \textit{for the Actual Flight Path} = \mathrm{A}\,(\mathrm{Xt}_1)^2 + \mathrm{B}\,\mathrm{Xt}_1 + 0$$

$$\mathrm{h}\ \ \textit{for the Straight Line} \ = \ \mathrm{G}_2\,\mathrm{Xt}_1$$

therefore: $\mathrm{A}\,(\mathrm{Xt}_1)^2 + \mathrm{B}\,\mathrm{Xt}_1 + 0 = \mathrm{G}_2\,\mathrm{Xt}_1$

Annex 1 - Height and Distance adjustment uncurl due to turns

$$A\,(Xt_1)^2 + B\,Xt_1 + 0 = G_2\,Xt_1$$

$$A\,(Xt_1)^2 + B\,Xt_1 = G_2\,Xt_1$$

$$A\,Xt_1 + B = G_2$$

$$B = G_2 - A\,Xt_1 \qquad \text{(Eq3)}$$

Assumption: The biggest error in height between the straight line and the Actual Flight Path, will be assumed as the actual error. *(conservative assumption)*

Premise: The biggest error in height "Δh_E" is expected to occur when the *slope* (gradient) of the Actual Flight path is equal to the *slope* (gradient) of the straight line. [the gradient of the straight line = "G_2"]

Premise: The slope (gradient) of the Actual Flight Path at any Xt location, is given by the derivative of the Actual Flight Path with respect to dXt

$$\textit{Actual Flight Path equation}:\ A\,(Xt)^2 + B\,Xt + 0 = h$$

$$\frac{d\,[A\,(Xt)^2 + B\,Xt + 0]}{dXt} = \frac{dh}{dXt}$$

$$2A\,(Xt) + B = \frac{dh}{dXt}$$

Remember the premise: The biggest error in height "Δh_E" is expected to occur when the *slope* (gradient) of the Actual Flight path is equal to the *slope* (gradient) of the straight line. [the gradient of the straight line = "G_2"]

$$2A\,(Xt) + B = \frac{dh}{dXt} = G_2$$

In consequence, we'll relabel "Xt" as "Xt_E", defined as the Xt location where the biggest error in height occurs.

$$2A\,(Xt_E) + B = G_2$$

$$2A\,(Xt_E) = G_2 - B$$

$$Xt_E = \frac{G_2 - B}{2A}$$

Using equation (Eq3) to substitute for the "B" term in the above equation:

$$Xt_E = \frac{G_2 - (G_2 - A\,Xt_1)}{2A}$$

$$Xt_E = \frac{G_2 - G_2 + A\,Xt_1}{2A}$$

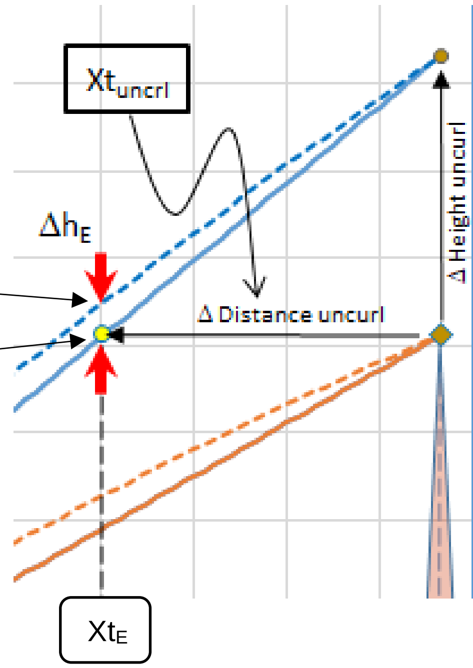
$$Xt_E = \frac{A\,Xt_1}{2A} = \frac{Xt_1}{2} \qquad \text{(Eq4)}$$

Annex 1 - Height and Distance adjustment uncurl due to turns

By having a definition Xt_E of where the biggest error in height occurs, the error in "h" (Δh_E) can be resolved as the difference between the "h_E" for the Actual Flight Path at that Xt_E location, minus the "h_E" for the Straight Line at that same Xt_E location.

$$h_E \; for\ the\ Actual\ Flight\ Path = A\,(Xt_E)^2 + B\,Xt_E + 0$$

$$h_E \; for\ the\ Straight\ Line \; = \; G_2\,Xt_E$$



$$\Delta h_E = \{A\,(Xt_E)^2 + B\,Xt_E + 0\} - \{G_2\,Xt_E\}$$

$$\Delta h_E = A\,(Xt_E)^2 + B\,Xt_E - G_2\,Xt_E$$

Using equation (Eq3) to substitute for the "B" term in the above equation:

$$\Delta h_E = A\,(Xt_E)^2 + (G_2 - A\,Xt_1)\,Xt_E - G_2\,Xt_E$$

$$\Delta h_E = A\,(Xt_E)^2 + G_2\,Xt_E - A\,Xt_1(Xt_E) - G_2\,Xt_E$$

The terms "$G_2\,Xt_E$" cancel out:

$$\Delta h_E = A\,(Xt_E)^2 - A\,Xt_1(Xt_E)$$

Using equation (Eq4) to substitute for the "Xt_E" term in the above equation:

$$\Delta h_E = A\left(\frac{Xt_1}{2}\right)^2 - A\,Xt_1\left(\frac{Xt_1}{2}\right)$$

$$\Delta h_E = A\,\frac{(Xt_1)^2}{4} - A\,\frac{(Xt_1)^2}{2}$$

$$\Delta h_E = \frac{A}{2}\left(\frac{1}{2}(Xt_1)^2 - (Xt_1)^2\right) = \frac{A}{2}(Xt_1)^2\left(\frac{1}{2} - 1\right) = \frac{A(Xt_1)^2}{2}\left(-\frac{1}{2}\right)$$

$$\boxed{\Delta h_E = \frac{-A(Xt_1)^2}{4}}$$