SAPOE

Society of Aircraft Performance and Operations Engineers

# SUMMARY OF USEFUL INFORMATION FOR PERFORMANCE AND OPERATIONS ENGINEERS

v 1.03



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**NOTE TO USERS**

This document is intended to serve members of the profession of aircraft performance and operations engineering.

It is a living document; it is our goal to keep it up to date as techniques and mathematical methods evolve in the fast-changing world of commercial aircraft.

If any errors are found in this material, we ask you to inform us immediately so they can be corrected.

Please help us keep improving the document for the benefit of all users. Any comments or suggestions on this document will be welcomed. Feel free to suggest material that could be added, text that can be improved: all comments and suggestions will be given serious consideration.

Please address any communications about this document to:

membership@sapoe.org.

We hope you'll find this summary to be useful.

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Notes:

Version 1.02 was released to correct an error in the equation of Mach number to calibrated airspeed on page 13.

Version 1.03 was released to correct an error in the equation of pressure ratio above the tropopause on page 6.

# INDEX

# ATMOSPHERIC PARAMETERS

In this section:

- The term ISA refers to the International Standard Atmosphere ("standard day") conditions.
- The tropopause is defined as occurring at 11000 meters exactly, or 36089.24 feet.
- The term ΔISA refers to the "ISA deviation", the temperature deviation from standard.

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## Outside air temperature (OAT)

Sea level standard day temperature:

$$T_0 = 15°C = 288.15°K = 59°F = 518.67°R$$

At or below the tropopause:

$$\text{OAT°C} = 15 - \left(0.0019812 \times h_p\right) + \Delta\text{ISA°C}$$

$$\text{OAT°K} = 288.15 - \left(0.0019812 \times h_p\right) + \Delta\text{ISA°C}$$

$$\text{OAT°F} = 59 - \left(0.00356616 \times h_p\right) + \Delta\text{ISA°F}$$

$$\text{OAT°F} = 518.67 - \left(0.00356616 \times h_p\right) + \Delta\text{ISA°F}$$

Where: $h_p$ is the pressure altitude in feet
ΔISA is the temperature deviation from standard value in degrees

Above the tropopause:

$$\text{OAT} = -56.5\ °C + \Delta\text{ISA}\ °C = 216.65°K + \Delta\text{ISA}\ °C$$

$$\text{OAT} = -69.7\ °F + \Delta\text{ISA}\ °C = 387.97°R + \Delta\text{ISA}\ °F$$

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## Temperature ratio θ

$$\theta = \frac{T}{T_0}$$

Where: T is the air temperature in absolute units, °K or °R
$T_0$ is the sea level standard day temperature in the same absolute units as T

$$\theta = \frac{\text{OAT°K}}{288.15} = \frac{\text{OAT°R}}{518.67} = \frac{\text{OAT°C} + 273.15}{288.15} = \frac{\text{OAT°F} + 459.67}{518.67}$$

## Temperature ratio θ (continued)

At or below the tropopause:

$$\theta = \frac{288.15 - (0.0019812 \times h_p) + \Delta\text{ISA°C}}{288.15}$$

Where: $h_p$ is the pressure altitude in feet

Or:

$$\theta = \frac{518.67 - (0.00356616 \times h_p) + \Delta\text{ISA°F}}{518.67}$$

Above the tropopause:

$$\theta = \frac{216.65 + \Delta\text{ISA°C}}{288.15} = \frac{389.97 + \Delta\text{ISA°F}}{518.67}$$

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## Pressure ratio δ

$$\delta = \frac{p}{p_0}$$

Where: p is the static pressure of the air in the same units as $p_0$
$p_0$ is the sea level standard day air pressure

$$p_0 = 29.92\ inches\ Hg = 14.696\ pounds\ per\ square\ inch = 1013.2\ hectoPascals$$

At or below the tropopause:

$$\delta = \left(\frac{288.15 - 0.0019812 \times h_p}{288.15}\right)^{5.25588} = (\theta_{ISA})^{5.25588}$$

Where: $h_p$ is the pressure altitude in feet

Above the tropopause:

$$\delta = 0.22336 \times e^{\left(\frac{36089.24 - h_p}{20805.7}\right)}$$

Where: e is the base of the natural logarithm, e=2.718281828

**Density ratio σ**

$$\sigma = \frac{\rho}{\rho_0}$$

Where: ρ is the density of the air in the same units as $\rho_0$
$\rho_0$ is the sea level standard day air density, 0.002377 slugs per cubic foot or equivalent

σ is normally found from ambient air pressure ratio δ and temperature ratio θ following the equation

$$\sigma = \frac{\delta}{\theta}$$

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**For convenience, a table of the International Standard Atmosphere parameters is provided at the end of this document.**

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## PRESSURE ALTITUDE

<u>Given the static air pressure:</u>

$$\delta = \frac{p}{p_0}$$

Where: p is the static air pressure in the same units as $p_0$
δ is the air pressure ratio

<u>Knowing the pressure ratio δ:</u>

At or below the tropopause (δ equal to or greater than 0.22336):

$$\mathrm{h}_p = 145442.15 \times (1 - \delta^{0.190263})$$

Where: $h_p$ is the pressure altitude in feet

Above the tropopause (δ less than 0.22336):

$$h_p = 36089.24 - 20805.7 \times \ln\left(\frac{\delta}{0.22336}\right)$$

## Pressure altitude (continued)

Given altimeter setting QNH and airport elevation:

$$h_p = \text{airport elevation} + 145442.15 \times \left[1 - \left(\frac{\text{QNH}}{p_0}\right)^{0.190263}\right]$$

Where: QNH is the reported altimeter setting for the airport in the same units as $p_0$
$p_0$ is the sea level standard day static pressure. 29.92 in. Hg or 1013.2 hPa

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# SPEED OF SOUND AND MACH NUMBER

## Speed of sound

In general:

$$a = \sqrt{\gamma RT}$$

Where: $\gamma$ is the ratio of specific heats ($C_P \div C_V$) and for air is equal to 1.4
a is the speed of sound in feet per second or, in metric units, meters per second
R is the specific gas constant:
R = 1716.5619 foot-pounds per slug-degree R in British units
R = 287.0529 Newton –meters per kilogram mass-degree K in metric units
T is the absolute temperature in degrees Rankine or, in metric units, degrees Kelvin

Speed of sound on a standard day at sea level:

$a_0$ is defined as the sea level standard day speed of sound

$a_0$ = 661.4786 knots

$a_0$ = 1116.45 feet per second

$a_0$ = 340.29 meters per second

For the speed of sound in feet per second:

$$a = a_0\sqrt{\theta} = 1116.45\sqrt{\theta}$$

Where: a is the speed of sound in feet per second
$a_0$ is the sea level standard day speed of sound in feet per second
$\Theta$ is the air temperature ratio

## Speed of sound (continued)

For the speed of sound in meters per second:

$$a = a_0\sqrt{\theta} = 340.29\,\sqrt{\theta}$$

Where: a is the speed of sound in meters per second
$a_0$ is the sea level standard day speed of sound in meters per second
Θ is the air temperature ratio

For the speed of sound in knots:

$$a = a_0\sqrt{\theta} = 661.4786\,\sqrt{\theta}$$

Where: a is the speed of sound in knots
$a_0$ is the sea level standard day speed of sound in knots
Θ is the air temperature ratio

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## Speed of sound ratio

$$\frac{a}{a_0} = \sqrt{\theta}$$

Where: a is the speed of sound in the same units as $a_0$
$a_0$ is the sea level standard day speed of sound

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## Mach number

In general:

Mach number is defined as true airspeed compared to the speed of sound for the given conditions $\frac{V_{true}}{a}$ stated as a decimal, e.g. Mach 0.84 signifying flight at a speed that is 84% of the speed of sound.

Mach number given the true airspeed and the air temperature ratio:

$$M = \frac{V_{true}}{a} = \frac{V_{true}(\text{knots})}{661.4786\sqrt{\theta}} = \frac{V_{true}(\text{ft/sec})}{1116.45\sqrt{\theta}} = \frac{V_{true}(\text{m/sec})}{340.29\sqrt{\theta}}$$

Where: θ is the air temperature ratio a/$a_0$
$V_{true}$ is the true airspeed

## Mach number (continued)

Mach number given the equivalent airspeed and the air pressure ratio:

$$M = \frac{V_{true}}{a} = \frac{V_{true}(\text{knots})}{661.4786\sqrt{\delta}} = \frac{V_{true}(\text{ft/sec})}{1116.45\sqrt{\delta}} = \frac{V_{true}(\text{m/sec})}{340.29\sqrt{\delta}}$$

Where: δ is the air pressure ratio $p/p_0$
$V_{true}$ is the true airspeed

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# TOTAL TEMPERATURE, PRESSURE, AND DENSITY

## Total air temperature (TAT or $T_{total}$)

$$TAT = OAT + (1 + 0.2M^2)$$

Where: OAT is the outside air temperature for the given conditions
M is the Mach number

NOTE: the given OAT must be in units of absolute temperature °K or °R, yielding a TAT in the same units of absolute temperature.

**For convenience, a table of total temperature for standard day temperatures at different altitudes and Mach numbers is provided at the end of this document.**

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## Total temperature ratio $\theta_{total}$

$$\theta_{total} = \theta\,(1 + 0.2M^2)$$

Where: θ is the air temperature ratio for the given conditions
M is the Mach number

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## Total pressure $p_{total}$

$$p_{total} = p_{static}\,(1 + 0.2M^2)^{3.5}$$

Where: $p_{static}$ is the static air pressure for the given conditions
M is the Mach number

## Total pressure ratio $p_{total}$

$$\delta_{total} = \delta\ (1 + 0.2M^2)^{3.5}$$

Where: δ is the static air pressure ratio for the given conditions
M is the Mach number

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## Total density $\rho_{total}$

$$\rho_{total} = \rho\ (1 + 0.2M^2)^{2.5}$$

Where: ρ is the static air pressure for the given conditions
M is the Mach number

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## Total density ratio $\sigma_{total}$

$$\sigma_{total} = \sigma\ (1 + 0.2M^2)^{2.5}$$

Where: σ is the density ratio for the given conditions
M is the Mach number

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# AIRSPEED AND MACH NUMBER CONVERSIONS

## Calibrated airspeed to equivalent airspeed

Given pressure altitude and calibrated airspeed

$$V_e = 1479.1 \sqrt{\delta \left[ \left( \frac{1}{\delta} \left\{ \left[ 1 + 0.2 \left( \frac{V_C}{661.4786} \right)^2 \right]^{3.5} - 1 \right\} + 1 \right)^{\frac{1}{3.5}} - 1 \right]}$$

Where: δ is the air pressure ratio for the known pressure altitude
$V_C$ is the calibrated airspeed in knots

<u>OR:</u> given the compressibility correction $\Delta V_C$ (from a chart, for example) and the calibrated airspeed, then:

$$V_e = V_C - \Delta V_c$$

**For convenience, a graph of $\Delta V_C$ as a function of $V_C$, altitude, and Mach number is provided at the end of this document.**

## Calibrated airspeed to Mach number

Given pressure altitude and calibrated airspeed

$$M = \sqrt{5\left[\left(\frac{1}{\delta}\left\{\left[1 + 0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

Where: M is the Mach number
δ is the air pressure ratio for the known pressure altitude
$V_C$ is the calibrated airspeed in knots

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## Calibrated airspeed to true airspeed

Given pressure altitude and air temperature

$$V_{true} = 1479.1\sqrt{\theta\left[\left(\frac{1}{\delta}\left\{\left[1 + 0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

Where: $V_{true}$ is the true airspeed in knots
$V_C$ is the calibrated airspeed in knots
Θ is the air temperature ratio
δ is the air pressure ratio

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## Equivalent airspeed to calibrated airspeed

Given pressure altitude and equivalent airspeed

$$V_C = 1479.1\sqrt{\left[\left(\delta\left\{\left[1 + \frac{1}{\delta}\left(\frac{V_e}{1479.1}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

Where: $V_e$ is the equivalent airspeed in knots
$V_C$ is the calibrated airspeed in knots
δ is the air pressure ratio

## Equivalent airspeed to Mach number

Given pressure altitude and equivalent airspeed

$$M = \frac{V_e}{661.4786}\sqrt{\frac{1}{\delta}}$$

Where: $V_e$ is the equivalent airspeed in knots
M is the Mach number
δ is the air pressure ratio

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## Equivalent airspeed to true airspeed

Given pressure altitude, air temperature, and equivalent airspeed

$$V_{true} = \frac{V_e}{\sqrt{\sigma}} = V_e\sqrt{\frac{\theta}{\delta}}$$

Where: $V_e$ is the equivalent airspeed in knots
$V_{true}$ is the true airspeed in knots
σ is the air density ratio
Θ is the air temperature ratio
δ is the air pressure ratio

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## Mach number to calibrated airspeed

Given Mach number and pressure altitude

$$V_C = 1479.1\sqrt{\left(\{\delta[(0.2M^2+1)^{3.5}-1]+1\}^{\frac{1}{3.5}}-1\right)}$$

Where: $V_C$ is the calibrated airspeed in knots
M is the Mach number
δ is the air pressure ratio

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## Mach number to equivalent airspeed

Given Mach number and pressure altitude

$$V_e = 661.4786 \times \mathrm{M}\sqrt{\delta}$$

Where: $V_e$ is the equivalent airspeed in knots
M is the Mach number
δ is the air pressure ratio

## Mach number to true airspeed

Given the air temperature

$$V_{true} = 661.4786 \times M\sqrt{\theta}$$

Where: $V_{true}$ is the true airspeed in knots
M is the Mach number
θ is the air temperature ratio

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## True airspeed to calibrated airspeed

Given the pressure altitude, air temperature, and true airspeed

$$V_C = 1479.1\sqrt{\left[\left(\delta\left\{\left[1+\frac{1}{\theta}\left(\frac{V_{true}}{1479.1}\right)^2\right]^{3.5}-1\right\}+1\right)^{\frac{1}{3.5}}-1\right]}$$

Where: $V_C$ is the calibrated airspeed in knots
$V_{true}$ is the true airspeed in knots
θ is the air temperature ratio
δ is the air pressure ratio

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## True airspeed to equivalent airspeed

Given the pressure altitude, air temperature, and true airspeed

$$V_e = V_{true}\sqrt{\sigma} = V_{true}\sqrt{\frac{\delta}{\theta}}$$

Where: $V_e$ is the equivalent airspeed in knots
$V_{true}$ is the true airspeed in knots
σ is the air density ratio
Θ is the air temperature ratio
δ is the air pressure ratio

## True airspeed to Mach number

Given air temperature and true airspeed

$$M = \frac{V_{true}}{661.4786\sqrt{\theta}}$$

Where: M is the Mach number
Vtrue is the true airspeed in knots
Θ is the air temperature ratio

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## Compressibility correction $\Delta V_C$

$$\Delta V_C = V_C - V_e$$

Where: $\Delta V_C$ is the airspeed correction due to compressibility, in knots
$V_C$ is the calibrated airspeed in knots
$V_e$ is the equivalent airspeed in knots

**For convenience, a graph of $\Delta V_C$ as a function of $V_C$, altitude, and Mach number is provided at the end of this document.**

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## Check case for calculations of airspeed conversions

Given: Flight level 350
OAT = ISA+10°C
M = 0.84

$$\theta = \frac{T}{T_0} = \frac{(288.15 - 0.0019812 \times 35000 + 10)}{288.15} = 0.7941$$

$$\delta = \theta_{ISA}{}^{5.25588} = \left(\frac{T_{ISA}}{T_0}\right)^{5.25588} = \left(\frac{288.15 - 0.0019812 \times 35000}{288.15}\right)^{5.25588} = 0.2353$$

$V_C$ = 287.1 knots

$V_e$ = 269.6 knots

$V_{true}$ = 495.2 knots

# DYNAMIC PRESSURE q

## Dynamic pressure

Given the true airspeed in feet per second:

$$q = \frac{1}{2}\rho V_{true}^2 = \frac{1}{2}(\rho_0 \times \sigma)\, V_{true}^2 = \frac{\sigma V_{true}^2}{841.1}$$

Where: $\rho$ is the air density in slugs per cubic foot
$\rho_0$ is the sea level standard air density in slugs per cubic foot
$\sigma$ is the air density ratio
$V_{true}$ is the true airspeed in feet per second

[Note: $841.4 = \frac{2}{0.002377}$ where 0.002377 is the sea level standard day density $\rho_0$ in slugs/ft$^3$ ]

Given the true airspeed in knots:

$$q = \frac{\sigma\, V_{true}^2}{296.369}$$

Where: $\sigma$ is the air density ratio
$V_{true}$ is the true airspeed in feet knots

[Note: 295.369 = $\frac{2}{0.002377 \times 1.6878}$ where 1.6878 is the velocity conversion from knots to feet per second ]

Given the equivalent airspeed in knots:

$$q = \frac{V_e^2}{295.369}$$

Where: $V_e$ is the equivalent airspeed in knots

( $\sigma\, V_{true}^2 = V_e^2$ )

Given the speed in Mach number:

$$q = 1481.4\, M^2 \delta$$

Where: M is the Mach number
$\Delta$ is the air pressure ratio

[Note: $1481.4 = \frac{661.4786^2}{295.369}$ where 661.4786 is the speed of sound at a sea level standard day in knots]

# LIFT AND DRAG COEFFICIENTS

## General expression for lift force

$$L = nW$$

Where: L is the lift force in the same units as W
n is the normal acceleration in g's; it is equal to 1.00 for level unaccelerated flight
W is the weight

## Lift and drag coefficients

<u>Given the airspeed in feet per second:</u>

$$C_L = \frac{L}{qS} = \frac{L}{\frac{1}{2}\rho V_{true}^2 S}$$

$$C_D = \frac{D}{qS} = \frac{D}{\frac{1}{2}\rho V_{true}^2 S}$$

Or:

$$C_L = \frac{841.4 \times L}{\sigma V_{true}^2 S}$$

$$C_D = \frac{841.4 \times D}{\sigma V_{true}^2 S}$$

Where: L is the lift force in pounds
D is the drag force in pounds
S is the reference wing area in square feet
q is the dynamic pressure in pounds per square foot
ρ is the air density in slugs per cubic foot
σ is the air density ratio

<u>Note:</u> in the previous two equations, the constant $841.4 = \frac{2}{0.002377}$ where 0.002377 is the sea level standard day density $\rho_0$ in slugs per cubic foot, and hence this form of the equations is valid only for forces in pounds and wing areas in square feet. Equivalent equations can be written for metric units.

## Lift and drag coefficients (continued)

Given the true airspeed in knots:

$$C_L = \frac{295.369 \times L}{\sigma\, V_{true}^2\, S}$$

$$C_D = \frac{295.369 \times D}{\sigma\, V_{true}^2\, S}$$

Given the equivalent airspeed in knots:

$$C_L = \frac{295.369 \times L}{V_e^2\, S}$$

$$C_D = \frac{295.369 \times D}{V_e^2\, S}$$

Given the speed in Mach number:

$$C_L = \frac{L}{1481.4\, M^2\, \delta\, S}$$

$$C_D = \frac{D}{1481.4\, M^2\, \delta\, S}$$

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## Engine-inoperative corrections to low-speed drag coefficient

$$C_{D_{engine-inop}} = C_{D_{all-engine}} + \Delta C_{D_{windmill}} + \Delta C_{D_{control}}$$

Where: $C_{D_{engine-inop}}$ is the drag coefficient of the airplane including the effects on drag of engine failure
$C_{D_{all-engine}}$ is the drag coefficient of the airplane for the given conditions, all engines operating
$\Delta C_{D_{windmill}}$ is the drag increment due to the windmilling and spillage drag of the failed engine
$\Delta C_{D_{control}}$ is the drag increment due to the deflections of the flight controls necessitated by the thrust asymmetry

Note: This is a general form of the engine-inoperative drag coefficient. Different manufacturers may account in different ways for the effects of engine failure on total airplane drag in the engine-out condition. Check for the relevant information for the airplane in question.

# YAWING MOMENT COEFFICIENT

## Two-engine airplanes

<u>Given the true airspeed in feet per second:</u>

$$C_N = \left[\frac{\left(F_{N_1} - F_{N_2}\right) \times \ engine\ moment\ arm}{S\ q\ b}\right]$$

Where: $F_{N_1}$ is thrust of the left engine in pounds
$F_{N_2}$ is the thrust of the right engine in pounds
b is the wing span in feet
q is the dynamic pressure in pounds per square feet
S is the reference wing area in square feet

or:

$$C_N = \left[\frac{\left(F_{N_1} - F_{N_2}\right) \times \ engine\ moment\ arm}{\sigma\ V_{true}^2\ S\ b}\right]$$

Where: $V_{true}$ is the true airspeed in feet per second

<u>Given the true airspeed $V_{true}$ in knots:</u>

$$C_N = \left[\frac{295.369 \times \left(F_{N_1} - F_{N_2}\right) \times \ engine\ moment\ arm}{\sigma V_{true}^2\ S\ b}\right]$$

<u>Given the equivalent airspeed $V_e$ in knots:</u>

$$C_N = \left[\frac{295.369 \times \left(F_{N_1} - F_{N_2}\right) \times \ engine\ moment\ arm}{V_e^2 S\ b}\right]$$

<u>Given the speed in Mach number:</u>

$$C_N = \left[\frac{\left(F_{N_1} - F_{N_2}\right) \times \ engine\ moment\ arm}{1481.4\ M^2\ \delta\ S\ b}\right]$$

## Four-engine airplanes

$$C_N = \frac{1}{S\ q\ b} \times \left|\left[\left(F_{N_1} - F_{N_4}\right) \times outbd\ moment\ arm\ + \left(F_{N_2} - F_{N_3}\right) \times inboard\ moment\ arm\right]\right|$$

Where: $F_{N_1}$ and $F_{N_4}$ are the left and right outboard engine thrusts respectively
$F_{N_2}$ and $F_{N_3}$ are the left and right inboard engine thrusts respectively

# RADIUS OF THE EARTH

$$r_e = \sqrt{\frac{a^4 + b^4\, tan^2\varphi}{a^2 + b^2\, tan^2\varphi}}$$

Where: $r_e$ is the radius of the earth in feet
a is the radius of the earth at the equator: 20,925,780 feet
b is the radius of the earth at the poles: 20,855,636 feet
$\varphi$ is the latitude in degrees

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# GRAVITATIONAL ACCELERATION

## Standard value of g

$g_0 = 32.17405\ ft/sec^2 = 9.80655\ m/sec^2$ at latitude 45.5425 degrees, sea level

<u>Note:</u> in some documents, the standard value of g is shown as occurring at 45 degrees latitude

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## Standard g corrected for latitude (Lambert's equation)

Lambert's equation yields a value of standard g at sea level and any latitude, including the effects of the earth's oblateness, the density distribution of the earth, and the earth's rotation.

$$g_{\varphi,SL} = 32.17244 \times [1 - 2.6373 \times 10^{-3} \cos(2\varphi) + 5.9 \times 10^{-6}(\cos^2 2\varphi)]$$

Where: $g_{\varphi,SL}$ is the acceleration of gravity at any latitude in feet per second per second
$\varphi$ is the latitude in degrees

<u>Note:</u> the equation above is consistent with Engineering Sciences Data Unit document 77022. In some other documents, the equation is given as:

$$g_{\varphi,SL} = 32.17405 \times [1 - 2.6373 \times 10^{-3} \cos(2\varphi) + 5.9 \times 10^{-6}(\cos^2 2\varphi)]$$

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## Standard g corrected for latitude and altitude

$$g_{\varphi,z} = \left(g_{\varphi,SL} + \omega_e^2\, r_e\, cos^2\varphi\right) \times \left(\frac{r_e}{r_e + z}\right)^2 - \omega_e^2\, (r_e + z)\, cos^2\varphi$$

Where: $\omega_e$ is the earth's rotation rate: $7.29212 \times 10^{-5}$ radians per second
$R_e$ is the earth's radius in feet
z is the height above sea level in feet

## Correction to g for an airplane's velocity

$$g = g_{\varphi,z} + \Delta g_{centrifugal}$$

$$\Delta g_{centrifugal} = -\left[\frac{V_G^2}{(r_e + z)} + 2\,\omega\, V_G \cos\varphi \sin\chi\right]$$

Where: $V_G$ is the true speed *<u>over the ground</u>* <u>in feet per second</u>
$\chi$ is the true track angle of the flight path in degrees

<u>Note:</u> the equation above for $\Delta g_{centrifugal}$ is sometimes shown as two separate corrections. The first term of the equation is sometimes referred to as the centrifugal correction, and the second term may be referred to as the Coriolis correction. Whether applied as two separate corrections or one, the result is identical.

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# CLIMB PATH ANGLE (GRADIENT) AND RATE OF CLIMB

## All-engine climb path angle

$$\gamma = \sin^{-1}\left[\frac{\frac{T-D}{W}}{1+\frac{V}{g}\frac{dv}{dh}}\right]$$

Where: $\gamma$ is the climb path angle measured above the horizontal, in degrees
T is the total airplane thrust
D is the total airplane drag
W is the airplane weight, in the same units as the thrust and drag
$C_D$ is the airplane drag coefficient
$C_L$ is the airplane lift coefficient

$1 + \frac{V}{g}\frac{dv}{dh}$ is the "acceleration factor". Refer to the discussion below.

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## Engine-inoperative climb path angle

$$\gamma = \sin^{-1}\left[\frac{\frac{T-D_{WM}}{W} - \frac{C_D + \Delta C_{D_{control}}}{C_L}}{1+\frac{V}{g}\frac{dv}{dh}}\right]$$

Where: T is the thrust of the operating engine(s)
$D_{WM}$ is the drag of the inoperative engine ("windmilling drag")

$\Delta C_{D_{control}}$ is the drag increment due to the flight control deflections necessitated by the thrust asymmetry, also sometimes referred to as $\Delta C_{D_\chi}$

## Engine-inoperative climb path (continued)

ESSENTIAL NOTE:

Exact determination of a climb gradient is an iterative calculation: that is, in order to calculate the climb gradient, it is necessary to know the amount of lift that is being created by the airplane; however, in order to calculate the lift, it is necessary to know the climb gradient since (for equilibrium in the axis perpendicular to the flight path of the airplane) the lift must be equal to the weight of the airplane multiplied by the cosine of the climb gradient.

When working with small climb gradients it is satisfactorily accurate to assume that the cosine of the climb gradient is equal to unity ( $\cos \gamma = 1.00$) and thus that lift equals weight. In that case, the climb gradient equation shown above becomes

$$\gamma = \sin^{-1}\left[\frac{\frac{T}{W} - \frac{D}{L}}{1 + \frac{V}{g}\frac{dv}{dh}}\right] = \sin^{-1}\left[\frac{\frac{T}{W} - \frac{C_D}{C_L}}{1 + \frac{V}{g}\frac{dv}{dh}}\right]$$

This assumption facilitates calculation of the gradient since $C_L$ may be then calculated knowing the airplane's weight, wing area, air density, and velocity, and $C_D$ is found from the drag polar for the calculated coefficient of lift.

For larger angles of climb, for example all-engine climb gradients, best accuracy can be achieved by first *assuming* that weight equals lift, then calculating the corresponding climb gradient $\gamma$, and then *repeating* the calculation of gradient assuming that the lift is equal to the weight multiplied by the cosine of the gradient previously calculated; this yields a *new* climb gradient. Continue this iterative process until the gradient calculation yields a gradient that is equal to the gradient assumed when calculating the lift force.

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## Climb gradient

The climb gradient is customarily expressed in percent, thus:

$$\text{climb gradient } = 100 \times \tan\gamma$$

It is sometimes convenient to think of the gradient as the amount of height gained when traveling a known horizontal distance.

That gradient may be measured through the air (for example, the minimum required second segment gradient is a gradient measured through the air, and hence is not a function of wind velocity); gradient is , however, sometimes is measured over the ground, for example when computing a gradient required for obstacle clearance.

That is,

$$gradient = \frac{100 \times height\ gained}{air\ distance\ flown}$$

## Climb gradient (continued)

OR it may be

$$gradient = \frac{100 \times height\ gained}{ground\ distance\ flown}$$

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## Acceleration factor

An airplane being flown in climb by a pilot or an autoflight system is normally climbing at constant indicated (or calibrated) airspeed or constant Mach number. In such a climb, the airplane is actually accelerating inertially because the <u>true</u> airspeed at constant indicated/calibrated airspeed or Mach increases with decreasing air density experienced during climb.

Increasing true airspeed during climb has a negative effect on climb angle, because some of the engine thrust is being used to produce the acceleration following the law of F = Ma and hence is unavailable for producing climb.

The climb angle decrement is a function of $\left(\frac{V}{g}\frac{dV}{dh}\right)$ and the "acceleration factor" is defined as:

$$acceleration\ factor = \left(1 + \frac{V}{g}\frac{dV}{dh}\right)$$

Calculating the acceleration factor depends on the form of the airspeed known and the altitude, thus:

<u>For constant calibrated airspeed below the tropopause:</u>

$$\frac{V}{g}\frac{dV}{dh} = 0.7M^2\ \left[\phi - 0.190263\left(\frac{T_{ISA}}{T}\right)\right]$$

Where: M is the Mach number
$T_{ISA}$ is the standard day air temperature at the known altitude
T is the ambient temperature at that altitude, in the same units as $T_{ISA}$

<u>For constant calibrated airspeed above the tropopause:</u>

$$\frac{V}{g}\frac{dV}{dh} = 0.7M^2\phi$$

<u>Note:</u> In the two equations above,

$$\phi = \frac{[(1 + 0.2M^2)^{3.5} - 1]}{0.7M^2(1 + 0.2M^2)^{2.5}}$$

## Acceleration factor (continued)

For constant Mach number below the tropopause:

$$\frac{V}{g}\frac{dV}{dh} = -0.13318\, M^2 \left(\frac{T_{ISA}}{T}\right)$$

For constant Mach number above the tropopause:

$$\frac{V}{g}\frac{dV}{dh} = 0$$

**For a graph of acceleration factor for constant calibrated airspeed, see the graphs and tables at the end of this document.**

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## Rate of climb

In general:

$$\text{rate of climb R/C } = V_{true} \sin\gamma$$

where: $\gamma$ is the climb angle in degrees
$V_{true}$ is the true airspeed in climb

For airspeed in knots and rate of climb in feet per minute:

$$\text{rate of climb R/C } = 101.268 \times V_{true} \sin\gamma$$

[Note: 101.268 = 1.6878 × 60]

# TURNING FLIGHT

## Radius of turn R

$$R = \frac{V_{true}^2}{g \tan\phi}$$

Where: $\phi$ is the bank angle in degrees
$V_{true}$ is the true airspeed
$g$ is the local acceleration of gravity

R will be in units of feet if $V_{true}$ is in feet/sec and $g$ is in feet/sec$^2$
R will be in units of meters if $V_{true}$ is in meters/sec and $g$ is in meters/sec$^2$

## Radius of turn R (continued)

For true airspeed in knots and radius in feet:

$$R = 0.8854\left(\frac{V_{true}^2}{\tan\phi}\right)$$

[Note: 0.8854 = $1.6878^2$÷32.174]

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## Rate of turn

$$\frac{d\theta}{dt} = \frac{g\tan\phi}{V_{true}}$$

Where: $\frac{d\theta}{dt}$ is the rate of turn in radians per second
g is the local acceleration of gravity
$\phi$ is the bank angle

For true airspeed in knots and rate of turn in degrees per second:

$$\frac{d\theta}{dt} = \frac{1092.1\ \tan\phi}{V_{true}}$$

[Note: 1092.1 = (57.3×32.174)÷1.6878, where 57.3 is degrees per radian]

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## Angle of bank

For true airspeed in knots and radius in feet:

$$\phi = \tan^{-1}\left(\frac{0.08854\,V_{true}^2}{R}\right)$$

For true airspeed in knots and rate of turn in degrees per second:

$$\phi = \tan^{-1}\left(\frac{V\ \frac{d\theta}{dt}}{1092.1}\right)$$

## Normal acceleration in turning flight

$$n = \frac{1}{\cos\phi}$$

Where: n is the normal acceleration in gees
$\phi$ is the bank angle

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# TAKEOFF DISTANCES AND TIMES

A standard method for calculating takeoff distances and times is to use "step integration", meaning that the acceleration and deceleration segments of the takeoff path are divided into small increments of speed change, or "steps". For each of these steps, the parameters of the acceleration or deceleration such as thrust and drag are assumed to be constant – an acceptable assumption provided the increments are sufficiently small.

Using that assumption allows simple calculation of the distance and time for each of the small steps. The distances and the times for each of these steps are then summed to arrive at the total acceleration or deceleration distance or time.

## Acceleration distance

In general:

$$\Delta S_G = \frac{\Delta V \times (\bar{V}_{true} - V_{wind})}{\text{acceleration evaluated at } \bar{V}_{true}}$$

Where: $\Delta V$ is the speed increment used for the step
$\bar{V}_{true}$ is the average speed during the step
$V_{wind}$ is the wind velocity during the step, along the takeoff direction

Rate of acceleration during a step:

$$\text{acceleration} = \frac{g}{W}\left[T - \mu_R(W - L) - \phi W - D\right]$$

Where: W is the weight
L is the lift force generated by the wings
T is the thrust force generated by the engine(s)
$\mu_R(W - L)$ is the rolling friction force retarding the acceleration
$\phi$ is the runway slope, in radians, where an uphill slope is positive
D is the aerodynamic drag generated by the airplane

## Acceleration distance (continued)

Note: Runway slopes are usually expressed in percent – that is, the increase or decrease in runway elevation divided by the distance over which that elevation change occurs, then multiplied by one hundred to express the slope in percent. Because runway slopes are very small angles, for practical purposes the runway slope in radians is equal to the slope in percent divided by one hundred. Further, for the small slopes that are typical of runways, the sine of the runway slope is essentially equal to the runway slope expressed in radians.

Note: the equation for acceleration can also be shown as:

$$\text{acceleration} = g\left[\frac{T}{W} - \mu_R - \phi - (C_D - \mu_R C_L)\,\frac{q\,S}{W}\right]$$

where: $C_D$ is the drag coefficient
CL is the lift coefficient
$q$ is the dynamic pressure
$S$ is the reference wing area

Combining the two preceding equations:

$$\Delta S_G = \frac{\Delta V \times (\bar{V}_{true} - V_{wind})}{\frac{g}{W}\left[T - \mu_R(W - L) - \phi W - D\right]_{at\,\bar{V}_{true}}}$$

Where: $\Delta S_G$ is the incremental acceleration distance
$\Delta V$ is the incremental speed change (step)
$\bar{V}_{true}$ is the average true airspeed through the incremental speed step
$V_{wind}$ is the wind velocity
$g$ is the local acceleration of gravity through the incremental speed step
T is the total average thrust through the incremental speed step
$\mu_R$ is the rolling coefficient of friction

To find the incremental distance $\Delta S_G$ in feet, the airplane and wind velocities must be in feet per second, the acceleration of gravity must be in feet per second per second, the thrust and weight must be in pounds, the dynamic pressure must be in pounds per square foot, and the wing area must be in square feet.

For true airspeed and wind speed in knots, and distance in feet:

$$\Delta S_G = \Delta V \times \frac{2.84867 \times (\bar{V}_{true} - V_{wind})}{g\left[\frac{T}{W} - \mu_R - \phi - (C_D - \mu_R C_L)\,\frac{q\,S}{W}\right]_{at\,\bar{V}_{true}}}$$

[Note: $2.84867 = 1.6878^2$]

## Acceleration time

For true airspeed and wind speed in knots, and time in seconds:

$$\Delta T_G = \frac{1.6878 \times \Delta V}{g\left[\frac{T}{W} - \mu_R - \phi - (C_D - \mu_R C_L)\ \frac{q\,S}{W}\right]_{at\ \bar{V}_{true}}}$$

Where: $\Delta T_G$ is the incremental acceleration time

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## Flare distance

Note: For the purposes of this document, "flare distance" is defined as the distance from the point of liftoff from the runway up to the point at which the airplane achieves a height of 35 feet above the runway.

Methods for the calculation of takeoff distances may vary from one airplane manufacturer to another.

Refer to the manufacturer's documents for the exact method used in their calculations of takeoff distances.

$$S_{flare} = 1.6878 \times \left(\frac{V_R + V_2}{2} - V_{wind}\right) \times \Delta t$$

Where: $V_R$ is the rotation speed
$V_2$ is the speed at 35 feet
$V_{wind}$ is the wind velocity
$\Delta t$ is the flight test time

Note: a tailwind is negative, thus it adds to the term $\frac{V_R+V_2}{2}$ whereas a headwind is positive and thus subtracts from the term $\frac{V_R+V_2}{2}$

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## Deceleration distance

In general:

$$\Delta S_G = \frac{\Delta V \times (\bar{V}_{true} - V_{wind})}{\text{deceleration evaluated at } \bar{V}_{true}}$$

Where: $\Delta V$ is the speed increment used for the step
$\bar{V}_{true}$ is the average speed during the step
$V_{wind}$ is the wind velocity during the step, along the takeoff direction

## Deceleration distance (continued)

Rate of deceleration during a velocity step:

$$\text{deceleration} = \frac{g}{W}\left[T - \mu_B(W - L) - \phi W - D\right]$$

Where: W is the weight

T is the thrust force generated by the engine(s), reverse thrust being negative thrust

$\mu_B(W - L)$ is the airplane retarding force due to the wheel brakes

$\phi$ is the runway slope, in radians, where an uphill slope is positive

D is the aerodynamic drag generated by the airplane

Combining the two equations:

$$\Delta S_G = \frac{\Delta V \times (\bar{V}_{true} - V_{wind})}{\frac{g}{W}\left[T - \mu_B(W - L) - \phi W - D\right]_{at\ \bar{V}_{true}}}$$

For true airspeed and wind speed in knots, and distance in feet:

$$\Delta S_G = \Delta V \times \frac{2.84867 \times (\bar{V}_{true} - V_{wind})}{\frac{g}{W}\left[T - \mu_B(W - L) - \phi W - D\right]_{at\ \bar{V}_{true}}}$$

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## Deceleration time

$$\Delta T_G = \frac{1.6878 \times \Delta V}{\frac{g}{W}\left[T - \mu_B(W - L) - \phi W - D\right]_{at\ \bar{V}_{true}}}$$

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# LANDING DISTANCES

The landing deceleration distances are calculated using a step integration method, in a manner similar to that used for takeoff distances.

### Air distance

$$S_{air} = 1.6878\left(\frac{V_{touchdown} + V_{approach}}{2} - V_{wind}\right) \times \Delta t_{air}$$

Where: $\Delta t_{air}$ is the time from the threshold to touchdown, established by flight test

## Transition distance

$$S_{trans} = 1.6878\left(\frac{V_{touchdown} + V_{brakes\ on}}{2} - V_{wind}\right) \times \Delta t_{trans}$$

Where: $\Delta t_{trans}$ is the time from touchdown until the airplane is in the full stopping configuration
$V_{touchdown}$ is the speed at touchdown; its relationship to $V_{approach}$ is established by flight test

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## Stopping distance

$$\Delta S_G = \Delta V \times \frac{2.84867\ (\bar{V}_{true} - V_{wind})}{\frac{g}{W}\left[\, T - \mu_B(W - L) - \phi W - D\, \right]_{at\ \bar{V}_{true}}}$$

Where: $\bar{V}_{true}$ is the average true airspeed through the incremental deceleration speed step

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# TIRE HYDROPLANING SPEED

For the tire hydroplaning speed in knots:

$$\text{hydroplaning speed} = 9\sqrt{P}$$

where: P is the tire pressure in pounds per square inch

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# WIND VELOCITY VERSUS HEIGHT ABOVE THE GROUND

The wind velocities $V_{W_1}$ and $V_{W_2}$ at any two heights $h_1$ and $h_2$ above the ground may be approximated by the following relationship:

$$V_{W_1} = V_{W_2}\left(\frac{h_1}{h_2}\right)^{\frac{1}{7}}$$

# GREAT CIRCLE DISTANCE

$$D = 60 \times \cos^{-1} \{ [ \sin(lat_1) \times \sin(lat_2)] + [\cos(lat_1) \times \cos(lat_2) \times \cos(long_2 - long_1)] \}$$

Where: $lat_1$ and $long_1$ are the latitude and longitude respectively of the starting point
$Lat_2$ and $long_2$ are the latitude and longitude respectively of the ending point
D is the great circle distance in nautical miles

Note: the following sign conventions apply:
North latitudes are positive, south latitudes are negative
West longitudes are positive, east longitudes are negative

Check case:

Calculate the great circle distance: from San Francisco (N37°37.0', W122°23.0')
to Tokyo (N35°46.0', E140°23.0')

The distance = 4439.3 NM